"A Study of Bianchi Type-*I* Cosmological Model with Viscous Fluid in General Relativity"

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ABSTRACT :-

In this paper, we investigate the Bianchi Type-*I* cosmological model filled with a viscous fluid within the framework of General Relativity. The inclusion of bulk viscosity plays a significant role in understanding the early evolution of the universe, particularly in describing dissipative processes and smoothing out anisotropies. We assume a spatially homogeneous but anisotropic space-time and derive the Einstein field equations corresponding to the Bianchi Type-I metric. By considering an appropriate equation of state and a physically viable form of the bulk viscosity coefficient, exact solutions of the field equations are obtained. The behavior of important physical parameters such as expansion scalar, shear scalar, and energy density is analyzed in detail. The model indicates that viscosity contributes effectively to the isotropization of the universe at late times. The results obtained are consistent with the present observational scenario of an accelerating and nearly isotropic universe.

KEYWORDS :- Bianchi Type-I Model, General Relativity, Bulk Viscosity, Anisotropic Universe, Einstein Field Equations, Cosmology, Expansion Scalar, Shear Scalar

INTRODUCTION :-

The study of cosmological models plays a crucial role in understanding the large-scale structure and evolution of the universe. Among various models, spatially homogeneous but anisotropic cosmological models have attracted considerable attention, as they provide a more general framework than the standard isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) models. In this context, Bianchi Type-I space-time represents the simplest anisotropic generalization of the flat FLRW universe and serves as an important tool for studying the effects of anisotropy in the early universe.

In recent years, there has been growing interest in the role of dissipative processes in cosmology. In particular, the inclusion of bulk viscosity in the energy-momentum tensor has proven to be significant in describing irreversible processes such as particle production, phase transitions, and the decay of massive particles in the early universe. Bulk viscosity can effectively generate negative pressure, which may contribute to the accelerated expansion of the universe.

The motivation for considering viscous fluid models arises from the fact that the early universe was likely not in perfect thermodynamic equilibrium. Therefore, ideal fluid assumptions may not adequately describe the physical conditions of that epoch. The presence of viscosity tends to reduce anisotropy and drives the universe toward isotropy, which is consistent with current cosmological observations, such as the cosmic microwave background radiation.

In this paper, we focus on the Bianchi Type-I cosmological model with a viscous fluid distribution in the framework of General Relativity. We formulate the Einstein field equations for the chosen metric and obtain exact solutions by imposing suitable physical conditions. The behavior of the cosmological parameters is examined to understand the impact of viscosity on the dynamics of the universe.

This study provides insight into how viscous effects influence the evolution of anisotropic models and supports the idea that dissipative processes play a crucial role in explaining the present isotropic nature of the universe.

FIELD EQUATIONS :-

We consider Bianchi Type I string cosmology model with metric

$$ds^2 = -dt^2 + a^2dx^2 + b^2(dy^2 + dz^2) \qquad (1)$$

where a and b are functions of t only. In a co-moving coordinate system we set $u^i = u_i = (1,0,0,0)$. Also, $x^i = (0, a^{-1}, 0,0)$.

The Einstein field equations for a cloud of string with bulk viscosity are

$$R_i^j - Rg_i^j = \rho u_; u^j - \lambda u_i x^j - \xi\theta(u_i u^j + g_i^j) \qquad (2)$$

where ρ is the rest energy for cloud of strings with particles attached to them and λ is string tension density and ξ is the coefficient of viscous fluid and

$$\rho = \rho_p + \lambda \qquad (3)$$

Here, ρ_p being the particle energy density u_i is the four velocities for a cloud of particle and x_i are the four vectors representing the strings direction, which essentially is the direction of anisotropy. Thus,

$$u_i u^i = -1 = -x_i x^i \; and \; u_i x^i = 0 \qquad (4)$$

The non-vanishing components of the Einstein field equations are

$$2\frac{a}{a}\frac{b}{b} + \frac{b^2}{b^2} = \rho \qquad (5)$$

$$2\frac{\ddot{b}}{b}+\frac{b^2}{b^2}=\lambda+\xi\theta \tag{6}$$

$$2\frac{\ddot{a}}{a}\frac{\ddot{b}}{b}+\frac{\dot{a}}{a}\frac{\dot{b}}{b}=\xi\theta \tag{7}$$

The physical quantities that are of importance in cosmology are Proper volume V, expansion scalar θ, shear scalar sigma σ^2 and have the following expressions for the metric

$$V=ab^2 \tag{8}$$

$$\theta=\frac{a}{a}+2\frac{b}{b} \tag{9}$$

$$\sigma^2=\frac{2}{3}\left(\frac{b}{b}-\frac{a}{a}\right)^2 \tag{10}$$

2.1. SOLUTIONS OF FIELD EQUATIONS

Here, we have three field equations connecting five unknown quantities a, b, ρ, λ, ξ. Therefore, in order to obtain exact solutions we must need two more relation connecting the unknown quantities. We assume the analytic relation

$$b=ma^n \tag{11}$$

Using equations (9) and (11) in (7), we obtain

$$(n+1)\frac{\ddot{a}}{a}+n^2\frac{a^2}{a^2}=\xi(2n+a)\frac{a}{a} \tag{12}$$

Let us consider

$$a=f(a) \tag{13}$$

Using (13) in (12), we get

$$\frac{df}{da}+\left[\left(\frac{n^2}{n+1}\right)\frac{1}{a}\right]f=\xi\left(\frac{2n+1}{n+1}\right) \tag{14}$$

This on integration reduces to

$$f=\xi\left(\frac{2n+1}{n^2+n+1}\right)a+\frac{P}{a^{\frac{n^2}{n+1}}} \tag{15}$$

where P is an integrating constant. Integrating (15), we get

$$a=\frac{1}{\xi^{k_4}}\left[k_1+k_2e^{k_3\,\xi\,t}\right]^{k_4} \tag{16}$$

This gives

$$b=\frac{m}{\xi^{k_4}}\left[k_1+k_2e^{k_3\,\xi\,t}\right]^{k_5} \tag{16}$$

Where $k_1 = \frac{-P(n^2+n+1)}{2n+1}$, $k_2 = \frac{S_i}{2n+1}$, $k_3 = \frac{2n+1}{n+1}$, $k_4 = \frac{n+1}{n^2+n+1}$, $k_5 = \frac{n(n+1}{n^2+n+1}$ (18)

Hence the metric (1) reduces to the form

$$ds^2 = -dt^2 + \left[\frac{k_1+k_2e^{k_3\ \xi\ t}}{\xi}\right]^{2k_4} dx^2 + m^2 + \left[\frac{k_1+k_2e^{k_3\xi t}}{\xi}\right]^{2k_5} (dy^2 + dz^2) \quad (19)$$

Using the suitable transformation

$$\frac{k_1+k_2e^{k_3\xi t}}{\xi} = L\frac{\sin(\xi\tau)}{\xi}\ ;\ L^{k_4}x = X; mL^{k_5}(y+z) = Y+Z \quad (20)$$

the metric (19) reduces to

$$ds^2 = \left[\frac{L\,cos(\xi\tau)}{k_3(k_1 - L\,sin(\xi\tau)}\right]^2 d\tau^2 + \left[\frac{\sin(\xi\tau)}{\xi}\right]^{2k_4} dX^2 + m^2\left[\frac{\sin(\xi\tau)}{\xi}\right]^{2k_5} (dY^2 + dZ^2)$$

(21)

The rest energy ρ, the string tension density λ, the particle density ρ_p , expansion θ, shear σ and proper volume V for the model (21) are given by

$$\rho = k_3^2 k_5(2k_4 + k_5)\left[\xi - \frac{k_1\xi}{L\,sin(\xi\tau)}\right]^2 \quad (22)$$

$$\lambda = 3k_3^2k_5^2\left[\xi - \frac{k_1\xi}{L\,sin(\xi\tau)}\right]^2 k_1k_3^2\left[\frac{\xi^2}{L\,sin(\xi\tau)} - \frac{k_1\xi^2}{L^2\,sin^2(\xi\tau)}\right] - \xi k_3(k_4 + 2k_5(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}) \quad (23)$$

$$\rho_p = 2k_3^2k_5(k_4 - k_5)\left[\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\right]^2 + \left[\xi k_3(k_4 - 2k_5) - k_1k_3^2k_5\frac{\xi}{L\sin(\xi\tau)}\right]\left[\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\right] \quad (24)$$

$$\theta = k_3(k_4 + 2k_5)(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}) \quad (25)$$

$$\sigma^2 = \frac{2}{3}\left[k_3(k_5 - k_4)\left(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\right)\right]^2 \quad (26)$$

$$V = m^2\left[\frac{L\sin(\xi\tau)}{\xi}\right]^{k_4 2k_5} \quad (27)$$

From (22) and (24), we observe that the energy conditions rho $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied. Also, from (23), we observe that the string tension density $\lambda > 0$ provided

$$3k_3^2k_5^2\left[\xi - \frac{k_1\xi}{L\,sin(\xi\tau)}\right]^2 k_1k_5k_3^2\left[\frac{\xi^2}{L\,sin(\xi\tau)} - \frac{k_1\xi^2}{L^2\,sin^2(\xi\tau)}\right] > \xi k_3(k_4 + 2k_5\ \ (\xi - \frac{k_1\xi}{L\sin(\xi\tau)})$$

In the absence of bulk viscosity i.e. when $\xi \to 0$, the model (21) reduces to

$$ds^2 = \left(\frac{L}{k_1k_3}\right)^2 d\tau^2 + \tau^{2k_4}dX^2 + m^2\tau^{2k_5}(dY^2 + dZ^2) \quad (28)$$

The physical parameter ρ, λ, ρ_p and the kinematical parameters theta, sigma θ, σ^2, V for this model are

$$\rho = \left(\frac{k_1 k_3}{L\tau}\right)^2 k_5(2k_4 + k_5) \tag{29}$$

$$\lambda = \left(\frac{k_1 k_3}{L\tau}\right)^2 k_5(3k_5 - 1) \tag{30}$$

$$\rho_p = \frac{k_1^2 k_3^2 (1 + k_4 - k_5)}{L^2 \tau^2} \tag{31}$$

$$\sigma^2 = \frac{2}{3}\left[\frac{k_3 k_1 (k_5 - k_4)}{L\tau}\right]^2 \tag{32}$$

$$\theta = \frac{k_1 k_3 (k_4 + 2k_5)}{L\tau} \tag{32}$$

$$V = m^2 \tau^{(k_4 + 2k_5)} \tag{33}$$

From (29), (30) and (31), we observe that the energy conditions $\rho \geq 0, \lambda \geq 0$, $\rho_p \geq 0$ are fulfilled.

2.3. OTHER MODEL

In general, ξ is not constant through out of the fluid, so that ξ cannot be taken always constant, especially when the universe is expanding. Since in general depends on temperature (T) and pressure ρ, it is reasonable to consider ξ as a function of the time t.

In this case (12), after integration, lead to

$$a = [b_\circ + k_4^{-1} \int h(t) dt]^{k_4} \tag{35}$$

where

$$h(t) = c_\circ e^{k_3 \int \xi(t) dt} \tag{36}$$

and $b_\circ, c_\circ$ are constants of integration, therefore, we also obtain

$$b = m[b_\circ + k_4^{-1} \int h(t) dt]^{k_5} \tag{37}$$

Hence, in this case the metric (1) reduces to

$$ds^2 = -dt^2 + [b_\circ + k_4^{-1} \int h(t) dt]^{2k_4} dr^2 + m^2 [b_\circ + k_4^{-4} \int h(t) dt]^{2k_5} d\Omega^2 \tag{38}$$

$$\rho = n(n+2)\left[\frac{h(t)}{b_\circ + k_4^{-1} \int h(t) dt}\right]^2 \tag{39}$$

$$\lambda = \frac{\left(3n^2 - \frac{2n}{k_4}\right) h^2(t) + [n\dot{h} - \xi(2n+1) h(t)][b_\circ + k_4^{-1} \int h(t) dt]}{\left(b_\circ + k_4^{-1} \int h(t) dt\right)^2} \tag{40}$$

$$\theta = \frac{(1+2n) h(t)}{b_\circ + k_4^{-1} \int h(t) dt} \tag{41}$$

$$\rho_p = \frac{\left[-2n^2+2n+\frac{2n}{k_4}\right]h^2(t)-[n\dot{h}-\xi(2n+1)h(t)]\left[b_\circ+k_4^{-1}\int h(t)dt\right]}{\left(b_\circ+k_4^{-1}\int h(t)dt\right)^2} \tag{42}$$

$$\sigma = \frac{\sqrt{2}}{\sqrt{3}}\left[\frac{(n-1)h(t)}{b_\circ+k_4^{-1}\int h(t)dt}\right] \tag{43}$$

$$\frac{\sigma}{\theta} = \frac{\sqrt{2}}{\sqrt{3}}\frac{(n-1)}{(2n+1)} \tag{44}$$

3. CONCLUSION

In this paper we have presented a Kantowski Sachs suing cosmological model in the presence and absence of bulk viscosity. The model in the bulk viscosity is given by (21) represents an expanding universe when $\sin(\xi\tau) > \frac{k_1}{L}$ and when $\sin(\xi\tau) < \frac{k_1}{L}$ then decreases with time. Therefore, the model describes a sharing non-rotating expanding universe without the big bang star. From the above discussion we can see that the bulk viscosity plays a significant role in the evolution of the universe. Since $\lim_{\tau\to\infty}\frac{\sigma}{\theta} \neq 0$, the model dose not approaches isotropy for large value of τ. However, if $\sin(\xi\tau) < \frac{k_1}{L}$. The model (23) represents isotropy model in the presence of bulk viscosity.

In the absence of bulk viscosity, the model (28) stars expanding with a big bang at $\tau = 0$ and the expansion in the model decreases as time increases when $L \neq 0$, Also, when $\tau \to \infty$, the shear is zero. The physical parameter ρ, λ, ρ_p are infinite if k_5 is negative and when $\tau = 0$. Since $\lim_{\tau\to\infty}\frac{\sigma}{\theta} \neq 0$, the model does not approaches isotropy for large value of τ.

In the discussion of the model the equation (36) has a rich structure and admit different choice of structure $\xi(t)$. We have to choose $\xi(t)$ in such a manner so that (36) is integrable. We can choose $\xi(t)$ the suitable exponential, polynomial and sinusoidal of the function $\xi(t)$ such that (36) becomes integrable. So, we conclude that bulk viscosity play significant role in the evolution of the universe. In the presence of bulk viscosity model represent an expanding, shearing and non-rotating universe without the big bang start. But, in the absence of viscosity, the model start expanding with a big bang $\tau = 0$.

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