

A Systematic Approaches of Multi-Objective Goal Programming in Decision Optimization

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Abstract: In the multiobjective allocation problem, there are p non-linear objective functions which later turn into soft goals with a single linear constraint (hard goal). To apply Chebyshev/Fuzzy goal programming approach, all the hard and soft goals must be in linear form so that the worst deviation from the approximated linear goals is minimized. We thus approximate the non-linear soft goals by linear ones and use the linearized soft goals for minimizing the worst deviation in finding the Chebyshev/Fuzzy point. The aspiration levels being used in the Chebyshev/Fuzzy goal programming approach are taken as the optimal values of the respective non-linear programming problems instead of those of the linear programming problems.

We can formulate the multiple character problems arising in the areas of Stratified Random Sampling, Two-Stage Sampling, Double Sampling and Response Errors as multiobjective convex programming problems with convex objective functions and linear constraints. This multiobjective problem is then solve by Chebyshev and Fuzzy goal programming approach.

Keywords: Multi-Objective Optimization, Multi-Objective Stratified Sampling, Mathematical Programming, Chebyshev's Goal Programming, Fuzzy Goal Programming,

I MULTIVARIATE STRATIFIED SAMPLING

Sampling theory deals with the problems associated with the selection of samples from a population according to certain probability mechanism. The purpose of survey is to obtain information about the population which is defined according to the aims and objects of the survey. Since the information on population is based on sample data, a stage is always reached in planning of a sample survey, at which a decision must be made about the size of the sample, size of the sampling unit, the sampling scheme, the scope of the survey, number of strata and stratum boundaries etc. These decisions have much significance, e.g., the decision regarding the size of sample to be selected is important because too large a sample implies a waste of resources and too small a sample diminishes the utility of the results obtained. The problem of deriving the maximum statistical information on a population characteristic has been formulated as an optimization problem by minimizing the cost of the survey subjected to the restriction that the loss of precision is within a certain prescribed limit or alternately by minimizing the loss in precision subject to the restriction that cost of the survey remains within the given budget.

Stratified sampling is the most popular among various sampling designs that are extensively used in sample surveys. The problem of determining the number of strata, the problem of cutting the stratum boundaries, the problem of optimum allocation of sample sizes to various strata are treated as optimization problems and solved by several authors.

In multivariate stratified sampling where more than one-population characteristics are to be measured on every selected unit, the above problems become more complicated because of the non availability of a single optimality criterion which is suitable for all the characteristics.

The problem of sample allocation in multivariate stratified sampling has drawn attention of researchers for long time starting apparently with Neyman (1934). It is felt that unless the strata variances for various character are distributed in the same way, the classical Neyman allocation based on the variances of a single character is of no use because an allocation which is optimum for one characteristic may not be acceptable for another. For this reason, there is no unique or even widely accepted solution to the problem of optimum allocation in multivariate stratified sampling. One way to resolve this problem is to search for a compromise allocation, which is in some sense optimum for all the characters.

The first author to give the convex programming formulation to the allocation problem in multivariate stratified sampling was Kokan (1963). Kokan and Khan (1967) derived an analytical solution to this convex programming problem. They also showed how the sample allocation problem in other designs such as two-stage sampling, double sampling and response errors can be viewed as a convex programming problem. Chatterjee (1968) also considered the allocation problem for multivariate stratified surveys.

The optimum allocation in multivariate stratified sampling using prior information about the population means within stratum can be obtained by assigning an L-variate normal prior distribution to the vector of within stratum population means, where L denotes number of strata. Ericsson (1965) stated the problem as to “minimize the posterior variance of the overall population mean subject to a total budgetary constraint”. He also discussed the case when more than one population characteristics are to be estimated, under the assumption that the strata are sufficiently similar with respect to the various characteristics. Soland (1967) also treated the case of multivariate stratified sampling when there is prior information concerning the unknown stratum means of all the variates. He discussed the stratification problem proposed by Dalenius (1953) and formulated it as a non-linear programming problem and also formulated other multivariate stratified sampling problems that may be solved by non-linear programming.

Ahsan and Khan (1977) considered the multivariate allocation problem where the prior information about the unknown within stratum means of p characters is available in terms of a multivariate normal distribution with known parameters.

Chaddha et.al. (1971) used dynamic programming technique to find the optimum allocation in univariate case. Omule (1985) used the same technique to obtain compromise allocation for multivariate case by minimizing the total cost of the survey when the sampling variances of the estimates of various characteristics are subjected to specified tolerances limits. Jahan et. al.(1994) applied the dynamic programming technique for obtaining the compromise allocation by minimizing the total relative increase in the variances as compared to the optimum allocation, when the costs for measuring the various characteristics are fixed in advance. Khan (1997) treated the multivariate problem as a multi-stage decision problem, in which the k-th stage of the solution provides the sample size for the k-th stratum.

Bethal (1989) expresses the optimal multi-character stratified sample allocation as a closed expression in terms of normalized lagrangian multipliers whereas Rahim (1994) proposed an alternative procedure based on distance function of the sampling errors of all the estimates. Various authors like Nandi and Aich (1995), Chernyak and Starytskyy (1998), Chernyak and Chornous (2000) either suggested new criteria or explored further the already existing criteria.

II OPTIMIZATION

The fundamental problem of optimization is to arrive at the best possible decision in any given set of circumstances. Of course, many situations arise where the best is unattainable for one reason or another; sometimes what is best for one person is worst for another; more often we are not at all sure what is meant by best. The first step, therefore, in mathematical optimization is to choose some quantity, typically a function of several variables, to be maximized or minimized, subject possibly to one or more constraints. The commonest type of constraints are equalities and inequalities which must be satisfied by the variables of the problem, but many other types of constraints are possible; for example a solution in integers may be required. The next step is to choose a mathematical method to solve the optimization problem; such methods are usually called optimization techniques or algorithms.

The theory and practice of optimization has developed rapidly since the advent of electronic computers in 1945. It came of age as a subject in the mathematical curriculum in the 1950's, when well established methods of the differential calculus and the calculus of variations were combined with the highly successful new techniques of mathematical programming which were being developed at that time.

The optimization problems that have been posed and solved in the recent years have tended to become more and more elaborate, not to say abstract. Perhaps the most outstanding example of the rapid development of optimization techniques occurred with the introduction of Dynamic programming by Bellman in 1957 and of the maximum principle by Pontryagin in 1958. The techniques were designed to solve the problems of the optimal control of dynamical systems.

The simply stated problem of maximizing or minimizing a given function of several variables attracted the attention of many mathematicians over the past fifty years or so for developing the solution techniques under mathematical programming.

III. MATHEMATICAL PROGRAMMING

A mathematical programming problem (MPP) can be stated as follows:

$$\text{Maximize (or minimize) } Z = f(x_1, x_2, \dots, x_n) \quad (3.1)$$

Subject to the constraints

$$g_i(x_1, x_2, \dots, x_n) \{ \leq, =, \geq \} 0; \quad i = 1, 2, \dots, m \quad (3.2)$$

$$\text{and} \quad x_j \geq 0; \quad j = 1, 2, \dots, n \quad (3.3)$$

where in (3.2) one and only one sign among $\{ \leq, =, \geq \}$ holds true for each i . Usually, unless specified otherwise, in an MPP all the involved functions are assumed to be continuously differentiable.

The variables x_j , $j = 1, 2, \dots, n$ are called decision variables, the function $Z = f(x_1, x_2, \dots, x_n)$ in (3.1) is called objective function, the conditions (3.2) are called the constraints and the additional restrictions in (3.3) are called non-negativity restrictions. Often (3.3) is also included in (3.2) and the MPP takes a more simple expression as:

$$\text{Maximize (or minimize) } f(x)$$

Subject to
$$g_i(\underline{x}) \{ \leq, =, \geq \} 0; \quad i = 1, 2, \dots, m$$

where $\underline{x}' = (x_1, x_2, \dots, x_n)$ is the vector of decision variables.

To develop the theory of mathematical programming either of the maximization or minimization problems may be taken as standard form because of the simple reason that maximization of $f(\underline{x})$ is equivalent to minimization of $-f(\underline{x})$ and vice-versa. Furthermore all the constraints can be described with \leq or $=$ or \geq by simple operations of multiplying by -1 and /or addition or subtraction of some slack or surplus variables defined to have a ≥ 0 value and noting that an equation is equivalent to two inequalities, one with \leq and the another with \geq sign. Thus we may transform any given MPP in the following form:

$$\left. \begin{array}{l} \text{Minimize } f(\underline{x}), \\ \text{Subject to } g_i(\underline{x}) \geq 0, \\ \text{and } \underline{x} \geq 0. \end{array} \right\} \quad (3.4)$$

Any \underline{x} satisfying the constraints and non-negativity restriction to an MPP is called a feasible solution to the MPP. The set of all feasible solutions to an MPP is usually denoted by F . Thus the set F for MPP (1.3.4) is $F = \{ \underline{x} \mid g(\underline{x}) \geq 0, \underline{x} \geq 0 \}$.

Any $\underline{x}^* \in F$ for which $f(\underline{x}^*) \leq f(\underline{x})$ for all $\underline{x} \in F$ is called an optimum solution for a minimization MPP.

The optimal value \underline{x}^* of the decision variables is the function of various parameters appearing in MPP, such as: the availability of resources, costs or profits and technological coefficients (coefficients of decision variables in constraint functions). If some or all of the parameters of an MPP are stochastic variables rather than deterministic quantities then the MPP is called a Stochastic Programming Problem.

If all the functions involved in an MPP are linear functions of decision variables the MPP is called a Linear Programming Problem (LLP). On the other hand if some or all the functions are nonlinear, the MPP is called a Nonlinear Programming Problem (NLPP).

Depending upon the nature of the involved functions, restrictions on the decision variables and the objectives function(s), an MPP (Linear and /or Nonlinear) can further be placed in one or more of the several classes such as Integer Programming Problem (IPP), Quadratic Programming Problem (QPP), Convex Programming Problem (CPP), Separable Programming Problem (SPP), Geometric Programming Problem (GPP) and Multiobjective Programming Problem (MOPP).

IV MULTIOBJECTIVE OPTIMIZATION

The single-objective approach had been so ignored, and so widely accepted, that it may seem hard to believe that it has only seen widespread use since 1947. Further, it is easy to forget the fact that in 1947 the very notion of even a single-objective function was considered quite revolutionary. Specifically, until the development of LP, the typical mathematical model consisted of either a system of equations or a system of inequalities and, for the most part,

one's attention was directed toward the determination of just a feasible solution (i.e., one that satisfied the system of constraints as opposed to one that both satisfied the constraint set and optimized a single measure of performance). As such, in 1947, the concept of the inclusion of an objective function was considered just as radical as some now view the inclusion of multiple-objective functions.

However, although the consideration of multiple objectives may seem a novel concept, virtually any nontrivial, real world problem invariably involves multiple objectives. For example, the success of an airplane is determined by such things as its cost (to be minimized), payload (to be maximized), speed (to be maximized), maximum range (to be maximized), weight (to be minimized), survivability (to be maximized) etc. And, in the design of an aircraft, we may actually hope to optimize each and every one of these parameters.

In the traditional LP model, each and every constraint is considered to be absolutely rigid. That is, if a solution does not satisfy each and every constraint it is termed infeasible. However, in real-world problems, the notion of strictly rigid constraints does not necessarily hold at least not for every constraint function. In real-world problems, we just may be able to tolerate a certain level of "violation" of a constraint. Such flexible constraints are termed "soft" constraints (or soft goals) and are frequently encountered when we deal with actual problems. Thus, a soft constraint is one that we would like to satisfy, but for which we would be able to accept some degree of "violation". On the other hand, a hard constraint (or hard goal) is one for which any degree of violation would be absolutely intolerable. However, from a traditional LP point of view, such notions as multiple objectives and soft constraints only serve to complicate the situation.

There are several ways in which the multiobjective problem might be modeled.

(i) Conversion to a linear program via objective function Transformation (or deletion)

The traditionalist would most likely decide that, regardless of what management may have stated, a single objective model is going to be employed. Thus, one way to force the problem into the single objective format is to select one of the objectives, use it as the single objective, and then either ignore the other objectives or treat them as (rigid) constraints.

(ii) Conversion to a linear program via Utility Theory:

A Method of Aggregation

Theoretically (and only theoretically), it should be possible to combine any number of objectives into an equivalent, single objective if we can determine a common measure of effectiveness (i.e., a so-called "Proxy") by means of which each of the objectives may be expressed. The basis of such an approach is the aggregation of multiple objectives into a single and, it can be considered, equivalent function.

(iii) Conversion to a Goal Program (GP)

When one employs utility theory, the bulk of one's efforts is typically dedicated toward obtaining an adequate and rational representation of the decision maker's (theoretically) preference function. However, when one uses goal programming the effort shifts toward that of obtaining a better representation of the actual problem, through the development of the

goal-programming model. Whichever approach is deemed “best” is strictly a function of one’s personal perspective.

There are actually a number of types of goal programs, each espousing a somewhat different philosophy (i.e, with respect to how to measure the “goodness” of a solution to a problem involving multiple, conflicting goals). Three of the most popular (as well as the most practical) forms of GP are Archimedean GP (i.e., weighted GP), Non-Archimedean GP (i.e., lexicographic GP, or preemptive GP), and Chebyshev GP (or Minimax GP, or Fuzzy GP).

To form a goal-programming model, the very first thing that must be done is to convert all objectives into goals. When we convert the objectives into goals, we apply the following guidelines:

A maximizing (or a minimizing) objective is converted into a type II (\geq) (or type I (\leq)) inequality by means of the establishment and inclusion of a right-hand side, or aspiration level value. Indeed, we convert a goal into a constraint. Specifically, in a typical model, some of the goals will be hard (i.e, they absolutely must be attained) and some will be soft (i.e., some deviation is tolerable). Thus, we need a means to indicate the deviations from the right-hand sides of the constraints corresponding to each goal, whether hard or soft. To accomplish this, we shall add negative deviations and subtract positive deviations from the left-hand sides of each goal (and constraint).

Now, although the model is expressed in terms of goals (where some are hard and some are soft), we next need a function by means of which the achievement of the minimization of the unwanted goal deviations may be measured. This function, in fact, is termed the goal programming achievement function. Further, we need a philosophy upon which to develop such a function. Two approaches are found in the literature.

(a) Archimedean Goal Programming

In Archimedean, or weighted, GP, we shall form an achievement function consisting of precisely two terms. The first term represents the sum of all unwanted deviations for those goals that are hard (i.e., the rigid constraints). The second is composed of the weighted sum of all unwanted deviations for those goals that are soft. Thus, the achievement function for the general Archimedean GP model is given as

$$\text{Lexmin } u = \left\{ \left(\mu^{(1)} \eta^{(1)} + \omega^{(1)} \rho^{(1)} \right) \left(\mu^{(2)} \eta^{(2)} + \omega^{(2)} \rho^{(2)} \right) \right\}$$

where

Lexmin= lexicographic minimum (or achievement function)

u = achievement vector (or achievement function)

$\eta^{(k)}$ = vector of negative deviations, at priority level k

$\rho^{(k)}$ = vector of positive deviations, at priority level k

$\mu^{(k)}$ = vector of weights for all negative deviations, at priority level k

$\omega^{(k)}$ = vector of weights for all positive deviations, at priority level k .

(b) Non-Archimedean Goal Programming

In non-Archimedean GP (also called lexicographic or preemptive GP) as well, we form an achievement function. However, the number of terms in this achievement function will always be three or more. As before, the first term represents the sum of all unwanted deviations for

those goals that are hard (i.e., rigid constraints). The second is composed of the weighted sum of all unwanted deviations for those goals at priority level two. The third is composed of the weighted sum of all unwanted deviations at priority level three, and so on. The general form of the achievement function for a non-Archimedean GP is given as

$$\text{Lexmin } u = \left\{ \left(\mu^{(1)} \eta^{(1)} + \omega^{(1)} \rho^{(1)} \right), \dots, \left(\mu^{(K)} \eta^{(K)} + \omega^{(K)} \rho^{(K)} \right) \right\}$$

Wherein the total number of priority levels is K (i.e., $k = 1, 2, \dots, K$).

A comprehensive presentation on goal programming and its extensions is given in Ignizio (1976), and a summary of different variations of goal programming is provided in Charnes and Cooper (1977). In addition, a wide survey of literature around goal programming up to the year 1983 is presented in Soyibo (1985).

The short comings and the solution of the goal programming were discussed by Khorramshahgol and Hooshiari (1991), Chakraborty and Sinha (1995), Chakraborty and Dubey (2001).

V CHEBYSHEV GOAL PROGRAMMING

Charnes and Cooper (1961) introduced the idea of Goal Programming. Later Charnes and Cooper (1977) discussed the solution of multiple objective optimization problems through Goal Programming (GP). Ignizio (1983, 1985) observed that the Chebyshev GP (or Minimax GP) and Fuzzy GP are (closely) related. Ignizio and Cavalier (1994) have illustrated the procedure of solving the multiobjective linear programming problem through an example by its formulation to Chebyshev Linear Goal Programming (LGP) and compared it by the Fuzzy LGP. They also discussed the Chebyshev multiplex model for solving multiobjective problem. The Minimax or Chebyshev formulation implies the optimization of a utility function where the maximum deviation is minimized. The underlying philosophy of Chebyshev LGP is to find that solution that serves to minimize the single worst unwanted deviation from any (soft) goal. This particular notion also provides the basis of what is called Minimax GP and Fuzzy programming or Fuzzy GP.

As with any GP approach, the first step is to convert the problem into one containing nothing but goals. Next, we solve the problem as a conventional LP, using but one objective at a time. Once we have solved such a problem, we have determined the best possible value of the objective being considered as an aspiration level. An aspiration level is employed in order to convert an objective into goal. It represents a target level for the given objective - a level that is desired and/or acceptable. The use of aspiration levels to transform objectives (which are to be optimized) into goals (which are to be achieved) is known as the concept of "Satisficing". Satisficing, in turn, is a pragmatic approach based upon the manner in which most organizations, and most individuals, approach real-world decision making (Simon, 1957 and March and Simon, 1958). That is, rather than attempting to achieve solution optimality (which is actually only meaningful for static, deterministic, error free, single objective problems), we hope to find a solution that comes "as close as possible" to satisfying our goals.

Consider the multiobjective linear programming problem

$$\begin{aligned} & \text{Minimize } Z_k, \quad k = 1, 2, \dots, p \\ & \text{Subject to } \underline{A}x (\leq, \geq, \text{ or } =) \underline{b} \\ & \quad x \geq \underline{0}. \end{aligned} \tag{5.1}$$

The general form of the Chebyshev LGP model may be written as

$$\begin{aligned}
 & \text{Minimize } \delta \\
 & \text{Subject to:} \\
 & \quad Z_k - \delta \leq L_k, \text{ for all } p \text{ objectives} \quad (5.2) \\
 & \quad \underline{Ax}(\leq, \geq, \text{or} =) \underline{b}, \text{ for all } m \text{ constraints} \\
 & \quad \delta \geq 0, \underline{x} \geq \underline{0}
 \end{aligned}$$

where

δ = dummy variable representing the worst deviation level

Z_k = a linear function representing the k^{th} objective

L_k = minimum value that Z_k can take on while solving the various LPs in (5.1) individually for Z_1, \dots, Z_p .

VI. FUZZY PROGRAMMING

Zimmermann (1978, 1981) developed fuzzy mathematical programming to solve the problems with several functions. Narasimhan (1980) in one of his papers discussed goal programming in fuzzy environment. Sandipan Gupta and Chakraborty (1997), use the fuzzy programming approach to multiobjective linear programming problems. Several other authors such as Kassem and Ammer (1996), Mohan and Nguyen (1999), Han-Lin Li and Chian-Son Yu (2000), Aghezzaf and Ouaderhman (2000) and Aghezzaf (2001) etc have also discussed the fuzzy programming approach for solving multiobjective fuzzy programming problems.

Like Chebyshev goal programming, the basis of fuzzy programming approach is also to minimize the worst deviation from any (soft) goal. Using Zimmermann's (1978, 1985) approach to fuzzy programming, and assuming that all objectives are of the minimizing type, we may represent the general fuzzy linear programming model as:

$$\text{Minimize } \delta \quad (6.1)$$

Subject to:

$$\frac{Z_k - L_k}{d_k} \leq \delta, \text{ for all } p \text{ objectives} \quad (6.2)$$

$$\underline{Ax}(\leq, \geq, \text{or} =) \underline{b}, \text{ for all } m \text{ constraints} \quad (6.3)$$

$$\delta \geq 0, \underline{x} \geq \underline{0} \quad (6.4)$$

where U_k = maximum value that Z_k can take on while solving the various LPs in (5.1) individually for Z_1, \dots, Z_p

L_k = minimum value that Z_k can take on while solving the various LPs in (5.1) individually for Z_1, \dots, Z_p .

$$d_k = U_k - L_k$$

and the left-hand side of (6.2) is termed the fuzzy membership function.

The purpose of the fuzzy goal programming approach is to find the solution that serves to minimize the largest fuzzy membership function [worst deviation level (δ)]. However the fuzzy programming model is identical to the Chebyshev programming model except for the weight given to δ .

VII. CONCLUSION

In the multiobjective allocation problem, there are p non-linear objective functions which later turn into soft goals with a single linear constraint (hard goal). To apply Chebyshev/Fuzzy goal programming approach, all the hard and soft goals must be in linear form so that the worst deviation from the approximated linear goals is minimized. We thus approximate the non-linear soft goals by linear ones and use the linearized soft goals for minimizing the worst deviation in finding the Chebyshev/Fuzzy point. The aspiration levels being used in the Chebyshev/Fuzzy goal programming approach are taken as the optimal values of the respective non-linear programming problems instead of those of the linear programming problems.

We can formulate the multiple character problems arising in the areas of Stratified Random Sampling, Two-Stage Sampling, Double Sampling and Response Errors as multiobjective convex programming problems with convex objective functions and linear constraints. This multiobjective problem is then solve by Chebyshev and Fuzzy goal programming approach.

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