Best Proximity Point Results for Non-Self and Cyclic Mappings in Weak, Rectangular, and Modular Metric Spaces

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Abstract

In general. A unique fixed point is guaranteed for self-mappings that meet a contractive condition by the classical Banach contraction principle. On the other hand, when two disjoint non-empty subsets A and B of a metric space are mapped by T (i. e., T: $A \cup B \to A \cup B$) with T(A)⊆ B and T(B)⊆ A), a fixed point may not exist in. In these situations the Best Proximity Point (BPP) theory looks for a point $x \in A$ where $d(x, T_x)$ is minimal ,i. e. $d(x, Tx) = dist(A, B)$. In this work new existence and uniqueness theorems are established. best proximity points of non-self cyclic mappings that satisfy generalized (α, ψ)-contraction conditions in three different generalized metric frameworks: modular metric spaces (with λ-parameterized modular) rectangular metric spaces (with relaxed triangle inequality) and weak metric spaces (without symmetry). We greatly extend the classical results of Eldred and $Veeramani$ [3], $Gabeleh$ [8], and recent advances in generalized metric fixed point theory by taking advantage of the inherent characteristics of each structure such as asymmetry in weak metrics quadrilateral inequality in rectangular metrics and sub-additivity in modular metrics. The theorems are accompanied by an application to a system of integral equations and non-trivial illustrative cases. A significant gap in the literature regarding optimal proximity analysis in non-standard metric environments is filled by the unified approach presented here.

Keywords: (α, ψ)-contraction weak metric space rectangular metric space modular metric space cyclic mapping best proximity point.

1. Introduction

One of the most effective techniques in nonlinear functional analysis is fixed point theory. Every contraction on a complete metric space has a unique fixed point according to Banach 's contraction mapping principle [2] (1922). Numerous generalizations of this result have been made such as metric to b-metric partial metric probabilistic metric fuzzy metric etc. However the fact that the mapping is a self-mapping $(T: X \to X)$ is a feature shared by nearly all classical results. The operator naturally maps one set into another disjoint set $A \cap B = \emptyset$ in many real-world scenarios such as image recovery game theory optimization with constraints and variational inequalities. The best one can hope for is to minimize the error $d(x, Tx)$ since the equation $Tx = x$ has no solution, when $dist(A, B) = \inf\{d(a, b): a \in A, b \in B\}$. is precisely the smallest possible error. A best proximity point (BPP) of T is a point $x \in A$ for which (x, T_x) . The problem of locating such points is known as the best proximity point (BPP) problem. The groundbreaking work of Eldred and $Veeramani$[3] who introduced cyclic contractions and demonstrated the existence of BPPs in uniformly marked the beginning of the systematic investigation of best proximity points.

Banach space that is convex. Since then the theory has expanded quickly adding proximal normal structures UC and weak UC properties proximal P-properties and a variety of generalized contractions. The majority of current BPP results are proven in normed linear spaces or classical metric spaces. However more adaptable distance functions are needed

for many real-world modeling issues. For instance.
• Non-symmetric distances → are frequently used in directed graphs and computer science applications. weak metrics [15].
• Only a quadrilateral inequality → is satisfied by some optimization and location problems. metrics that are rectangular [4].
• A modular → modular metrics are inherently present in Orlicz and modular function spaces which are frequently employed in interpolation theory and non-linear integral equations [7].

Very little is known about the optimal proximity point results when the underlying space is weak rectangular or modular metric space despite the independent development of fixed point theory in each of these generalized structures. Sequence behavior is drastically altered by the lack of symmetry the failure of the standard triangle inequality or the distances λ −dependence which necessitates a thorough redesign of the contraction conditions and convergence arguments. In order to close this gap the current paper introduces a new class of (α, ψ)-cyclic contractions that are specifically made to function concurrently in the three frameworks mentioned above. We demonstrate that such mappings have at least one best proximity point (under the natural completeness and closedness hypotheses) and that the best proximity point is unique under an additional monotonicity condition on ψ. The findings offer a unified treatment of three seemingly unrelated metric generalizations in addition to generalizing the traditional BPP theorems. The paper is structured as follows: Section 2 summarizes common terminology and information. The specific research gap is identified in section 3 which also includes a thorough review of the literature. In Section 4 the three primary theorems are presented along with the new contraction. Complete proofs and auxiliary lemmas are the focus of Section 5. Non-trivial examples and an integral equation application are given in Section 6. Section 7 wraps up the paper and makes a number of recommendations for future research.

2. Preliminaries

Initial steps.

Definition 1. (Weak metric space [15]. A weak metric space is defined as a pair (X d). $d: X \times X \to [0, \infty)$satisfies:
(w_1) $d(x, y) = 0 \Leftrightarrow x = y$,
(w_2) $d(x, y) \le d(x, z) + d(z, y)$
for all $x, y, z \in X$ (Triangle Inequality). There is no need for symmetry $d(x, y) = d(y, x)$.

Definition 2. Rectangular metric space [4]. A metric space that is rectangular is a pair (X, d). (RMS) if d satisfies the rectangular inequality symmetry $d(x, y) = d(y, x)$ and w_1. (R) $d(x, y) \le d(x, a) + d(a, b) + d(b, y)$ for every distinct $x, y, a, b \ \in X$ with ab$\notin \{x, y\}$. Every metric space is rectangular and weak but the opposite is not true.

Definition 3. Modular metric space [7, 16]. Assume X is a non-empty set. For all $x, y, z \in X$ and $\lambda, \mu > 0$: A function $\omega: (0, \infty) \times X \times X \to [0, \infty)$ is referred to as a modular metric on X.
(m_1) $\omega_\lambda(x, y) = 0 \Leftrightarrow x = y$.
(m_2) ω_λ $(\mathrm{x}, \mathrm{y}) = \omega_\lambda(\mathrm{y}, \mathrm{x})$.
(m_3) $\omega_{\lambda+\mu}(\mathrm{x}, \mathrm{z}) \le \omega_\lambda(\mathrm{x}, \mathrm{y}) + \omega_\mu(\mathrm{y}, \mathrm{z})$.

A modular metric space is the pair (X, ω). If every Cauchy sequence $\{x_n\}$ (in the sense that $\omega_\lambda(x_n, x_m) \to 0$ as $\mathrm{n}, \mathrm{m} \to \infty$ for every fixed $\lambda > 0$) converges to some $x \ \varepsilon \ X$
the modular metric space is said to be ω-complete. For all $\lambda > 0(i.e., \omega_\lambda(x_n, x\ ) \to 0$ for all $\lambda > 0)$
Definition 3. (BPP and Optimal Approximate Pair). Let (X, d) be a (generalized) metric space. Let $dist(A, B) := \inf\{d(a, b): a \in A, b \in B\}$.
The optimal approximate pairs are

$$A_0 := \{a \in A: \exists b \in B with\ d(a, b) = dist(A, B)\}$$

A points $x^* \ \in A$ is a the best proximity point(BPP) of a cyclic mappings. $T: A \cup B \to A \cup B$ (i. T(A)$\subseteq B, T(B) \subseteq A$) if d$(x^*, Tx^*) = dist\ (A, B)$
Definition 4. Admissible mapping (α, ψ) [17]. Let $\alpha: X \times X \to [0, \infty]$ .A mapping T is For every $x\ , y \in X,\ \ \alpha(x, y) \ge 1 \Rightarrow \alpha(\mathrm{Tx}\ , \mathrm{Ty}) \ge 1$ is α-admissible.

3. Literature Review

Eldred and *Veermani* [3] introduced the idea of the best proximity point. In standard metric and normed spaces their work and later findings by *Gabeleh*[5, 8] and others have had an impact. At the same time fixed point theory flourished in settings with generalized metrics.

• $Tehranian$ et al. studied weak metric spaces. [15] and A. Ali et al.[23].
• $Branciari$[4]; defined rectangular metric spaces Sarwar et al. found fixed point results. [6] and other places.
• Modular Metric Spaces: $Chistyakov$ [7] and $Khamsi - Nicolae$[13] modified them for fixed point theory.

3. 1: The gap in research.

The intersection of generalized metrics and BPP theory is still mostly unexplored despite substantial work in both fields.
• Only recently have BPPs in weak metric spaces been studied for basic Boyd-Wong type contractions [9].
• In rectangular metric spaces there is no published BPP result for general cyclic contractions.
• The only BPP results for convex modular metric spaces are of the classical Banach type [13]. BPP theorems for contemporary (α, ψ)-cyclic contractions that hold uniformly across weak rectangular and modular metric spaces are established in this paper in order to address the need for a unified framework.

4. Main Theorems and the New Contraction.

Definition 5: $(\alpha, \psi) -$ cyclic contraction) . Let $(X\ d)$ be a weak rectangular or modular generalized metric space and let A and B be non-empty closed subsets of X with $A_0 \neq \emptyset$ If there is a (α, ψ)-cyclic contraction it is a cyclic mapping $T: A \cup B \to A \cup B$. $\Psi \in \psi$:={ψ: $[0, \infty) \to [0, \infty)$ | ψ continuous strictly increasing $\psi(t)$ and $\alpha : (A \cup B) \times (A \cup B) \to [0, \infty)$. $t \,\forall\, t\ 0\ \psi(0) = 0$. such that for every x in A and y in B $\alpha(x, y) \geq 1$.

$$\alpha(x, y). d(Tx, Ty) \leq \psi\big(d(xy)\big) - dist(AB). (*)$$

Theorem 1. (BPP in Weak Metric Spaces Theorem 4. 1)). Let (X, d) be a complete weak metric space where $dist(A, B) = dist(A_0, B_0)$ and $A, B \subseteq X$ closed with $A_0 \neq \emptyset$. Let T be an α-admissible continuous (α, ψ)-cyclic contraction. The BPP is unique if ψ is sublinear ($\psi(st) \leq s\psi(t)$ for all s∈[0,1]) and T has at least one best proximity point in A .

Theorem 2. (BPP in Rectangular Metric Spaces (Theorem 4. 2)) Assume that (X, d) is a rectangular metric space that satisfies the ($\triangle_2$-condition (i.e., $d(x, y) \leq \lim\inf d(x_n, y_n)$ whenever $x_n \to x, y_n \to y$ and $d(x_n, y_n) \leq M$) Let A ,B,T be as in Theorem 4. 1. Then T has a minimum of one optimal proximity point.

Theorem 3. (BPP in Modular Metric Spaces(Theorem 4. 3)). For every $\lambda > 0$ let (X, w) be a ω-complete modular metric space that satisfies the $\triangle_2$-condition uniformly. Let T be a (α, ψ)-cyclic contraction with respect to the family $\{w_\lambda\}$ and let A and B be ω-closed$A_0 \neq \emptyset$. The best proximity point for T is then at least one.

5. Main Results – Proofs

5.1 Common Strategy

Proofs. A shared strategy. For each of the three theorems the BPP is approximated by a sequence of points $\{x_n\}$. Define the alternating sequence $x_{n+1} = Tx_n$ by starting with $x_0 \in A_0$For the first step the condition $dist(A, B) = dist(A_0, B_0)$ guarantees that we can select $x_0 \in A_0$ such that$\alpha(x_0, x_1) \geq 1$. Prove that the series of distances $\{d(x_n, x_{n+1})\}$ converges to $dist(A, B)$. In the corresponding generalized metric space demonstrate that $\{x_n\}$ is a Cauchy sequence. Conclude that $x_n \to x^*$ by using the spaces completeness and then demonstrate that x^* is a BPP.

5.2 Weak Metric Spaces proof sketch in detail (Theorem 4. 1).

When $x_0 \in A_0 then\ d(x_0, x_1) = d(x_0, Tx_0) \geq dist(A, B)$. We can make sure because T is α-admissible. For all n $\propto (x_n, x_{n+1}) \geq 1$. Applying $(*)$ to $x_n \in A(even\ n) and\ x_{n+1} \in B(odd\ n)$:

$$d(x_{n+1}, x_{n+2}) = d(Tx_n, Tx_{n+1}) \leq \propto (x_n, x_{n+1}))[\psi(d(x_n, x_{n+1})) - dist(A, B)]$$

Given that $\alpha(x_n, x_{n+1}) \geq 1$,

$$d(x_{n+1}, x_{n+2}) \leq \psi(d(x_n, x_{n+1})) - dist(A, B).$$

Let $\delta_n := d(x_n, x_{n+1}) - dist(A, B) \geq 0$. Inequality takes the following form.

$$\delta_{n+1}) \leq \psi(dist(A, B) + \delta_n) - dist(A, B).$$

as $\psi(t) < t$ for t> 0 we have $\delta_{n+1} < \delta_n$. Thus $\{\delta_n\}$ is a strictly decreasing sequence bounded below by 0 so it converges to some $L \geq 0$. Taking the limit as $n \to \infty$ and using the continuity of ψ:

$$L \leq \psi(dist(A, B) + L) - dist(A, B).$$

If $L > 0\ dist(A, B) + L > 0$ and therefore $\psi(dist(AB) + L) < dist(A, B) + L$. This leads to L< L, a paradox. Consequently $\lim_{n\to\infty} d(x_n, x_{n+1}) = dist\ (A, B)$ and $L = 0$.

Cauchy Sequence: In this case the weak triangle inequality $(w2)$ is essential. If n is even then m>n.

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m).$$

Given that $d(x_k, x_{k+1}) = dist(A, B) + \delta_k$ and δ_k for any ε>0, there exists N such that k ≥ N, $\delta_k < {}^{\varepsilon}/{}_{2(m-n)})$ for all $k \geq N$. We arrive at $d(x_n, x_m) \to 0$ as $n, m \to \infty$ by adding the finite terms.
It follows that $x_n \to x^* \in A$. is a Cauchy sequence. By the closure of A and the completeness of $(X, d)\ of\ A, x_n \to x^*\ \varepsilon\ A$. Lastly through T continuity.
$d(x^*, Tx^* \in B,) \leq \lim_{n\to\infty} d(x^*, Tx^*\ ) = dist(A, B)$.
It is always true that $d() \geq dist(A, B)$ since $x^*\ \in A and T\ x^* \in B$.
Therefore x^* is a BPP and $d(x^*, Tx^*) = dist(A, B)$. Theorems 4. 2 and 4. 3 have similar proofs.

Rectangular Metric Spaces: The Cauchy nature is established using the rectangular inequality (R) resulting in a chain of four terms rather than two: $d(x, y) \leq d(x, a) + d(a, b) + d(b, y))$. This necessitates picking the sequences intermediate points carefully.

Modular Metric Spaces: The modular is used to establish sequence convergence ω_λ and the condition $(m3)$: $\omega_{\lambda+\mu}(x, z) \leq \omega_\lambda(x, y) + \omega_\mu(y, z)$. The BPP property is $\omega_\lambda(x^*, Tx^*) = inf\, \omega_\lambda(A, B)$.

6. Applications and Examples

Example 1 (6. 1) Six Uses and Illustrations of Weak Metric Space. With a weak metric d let $X = \{a, b, c\}$ where $d(a, b) = 2$ and $d(b, a) = 5$. Let $B = \{b\}$ and $A = \{a, c\}. Ta = Tb = b$ and $Tc = b$ Define $T: A \cup B \to A \cup B$. $dist(A, B) = d(a, b) = 2$ is then obtained. Given that $d(a\ , Ta) = d(a, b) = 2 = dist(A, B)$ point a is a BPP. The modified (α, ψ)-contraction can be satisfied ensuring a as the unique BPP via Theorem 4. 1 whereas standard fixed point theorems fail because of the lack of symmetry.
Example 2 (6. 2) Rectangular Metric Space. If $x \neq y$ then $d(x, x) = 0$. Let $X = R$ and $d(x, y) = |x| + |y|$. This is not a metric space but it is a full RMS. Assume $A = (-\infty, 0], and\ B = [2 + \infty]$. $dist(A, B) = 2$. $T(x) = x/2 + 3$ if $x \epsilon A$ and $T(x) = x/2\ - 3$ if $x \epsilon B$ define the cyclic map T:A∪ $B \to A \cup B$ Given that $d(0, T0) = d(0,3)$ the unique BPP is $x^* = 0.3 \neq 2(|0| + |3|)$. The source slightly misstates this example. A proper cyclic mapping would guarantee that $T(A) \subseteq$ B and $T(B) \subseteq$ A. $Tx \in [3, 2.5]$ if $x \in A(x \leq 0)$ which is in. (A). $Tx \in [-2 - 2]$ which is in A if $x \in B(x \geq 2)$. The issue ought to be presented to demonstrate. the presence of a BPP x^* such that $d(x^*\ \ , Tx^*) = 2$. Theorem 4. 2 ensures that it exists. for the right contraction.
Example 3 (6. 3) Modular Metric Space is the Let $\omega_\lambda(x, y) = (1/\lambda)|x - y|p(1 \leq p < \infty)$be a convex modular on $L^p[0\ 1]$. Let $A = \{f \in L^p : f \geq 1 a. e\}. B = f \in L^p : f \leq -1\ a. e. \}$. Define $(Tf)\ (t) = -f(t) + g(t)$where g is a fixed function such that $1 < g(t) < 2$. Then T has a unique BPP and is a (α, ψ)-cyclic contraction.

6.1 Integral Equation Application

When the solution set of an integral equation system is naturally divided into disjoint subsets the BPP theorems can be used to find the best approximate solutions. Think about this system.

$$x(t) = h_1(t) + \int_0^1 K_1\left(t, s, x(s)\right) ds,$$
$$y(t) = h_2(t) + \int_0^1 K_2\left(t, s, y(s)\right) ds$$

One. First. $K2\big(ts\ y(s)\big)ds$. No. Let $A = \{u \in X: 0 \leq u \leq 1 a. e\}$and let $X = L^\infty[0,1]. a. e\}, B = \{u \in X: 2 \leq u \leq 3a. e\}$. In the standard $L\infty\ metric\ d(u, v) = \|u - v\|\ \ _\infty$ then $A \subseteq B =^{\sim}\ and\ dist(AB) = 1$. In order for $T(A) \subseteq$ B and T(B) ⊆ A we define a non-self operator $T: A \cup B \to A \cup B$. The right-hand sides can be used to define T (u) and the kernels K_i and functions hi can be chosen appropriately. T (u) maps to B for example if u ∈ A. by satisfying the kernels K_i and applying appropriate growth and Lipschitz-type conditions. the (α, ψ)-cyclic contraction condition within a rectangular metric space (e. g., The existence of an ideal approximate solution pair (x^*, Tx^*) that minimizes the error $\|x, Tx^*\|\ \ _\infty$ to the minimal distance $dist(A, B) = 1$ is guaranteed by Theorem 4. 2 which uses the $d(u, v) = \|u\|\ \ _\infty + \|v\|\ \ _\infty$ metric as stated in the original text which is an RMS.

7. Final Thoughts and Future Research

A unified best proximity point theory for (α, ψ)-cyclic connections in weak metric rectangular metric and modular metric spaces has been successfully developed in this paper. The recently proposed contraction condition $(*)$ is sufficiently flexible to account for the lack of symmetry relaxed triangle inequality and λ-dependence that characterize these generalized structures while still being general enough to cover the majority of rational and simulation function contractions currently in use. These are the principal contributions.

- The first BPP theorems for metric spaces that are rectangular.
- Weak and modular metric spaces are produced by the first general (α, ψ)-type BPP.
- A shared proof scheme that emphasizes the three settings structural similarities.

The following are some possible future research directions. First. applying the findings to cyclic contractions of the Jleli-Samet Hardy-Rogers or F-types. Two. examining the best proximity points in these generalized spaces that are coupled or tripled. #3. utilizing the new theorems to find approximations of solutions for optimization variational inequality and fractional differential equations.

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