CORRELATION MEASURE FOR INTUITIONISTIC FUZZY MULTI NUMBERS

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Abstract: For the Triangular, Trapezoidal, Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal, and Decagonal Intuitionistic Fuzzy Multi Numbers, the Correlation Measure is proposed in this paper. This concept is an extension of correlation measures for Intuitionistic Fuzzy Multi sets which enables the quantification of the degree of association between attributes represented with Intuitionistic Fuzzy uncertainty. The properties of the proposed correlation measure are verified, and numerical examples are presented to demonstrate its applicability and efficiency in modelling complex uncertain information. This measure provides a valuable tool for decision-making in environments with imprecise and hesitant information.

Keywords: Correlation Measure, Fuzzy Numbers, Intuitionistic Fuzzy Numbers, Triangular, Trapezoidal, Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal, and Decagonal Intuitionistic Fuzzy Multi Numbers.

1. INTRODUCTION

In real life situation, information is often uncertain and incomplete to handle mathematically. To study such uncertainly **Zadeh (1965) [12]** introduced fuzzy set, an element which can have a membership value between 0 and 1. Thus, the fuzzy concept helps to represent uncertain or unclear information but in many real-life situations a single membership value is not enough. That is, sometimes a person may agree to a statement, disagree with it and still feel unsure at the same time. To handle this kind of situation, **Atanassov (1986,1989) [1,2]** introduced intuitionistic fuzzy sets (IFS). In an IFS, every element has three components namely membership degree (μ), non-membership (γ) and hesitation degree ($\pi = 1 - \mu - \gamma$). And this helps to describe human uncertainty more clearly. However, many real-life decisions involve multiple opinions multiple criteria or multiple observations for the same element. In such cases, having just one membership and one non-membership value may not be enough. To represent this type of multi-level uncertainty, Fuzzy Multi Number (FMN) and Intuitionistic Fuzzy Multi Number (IFMN) were developed by **Vivek et.al [8,9,10]**

An IFMN allows more than one membership and non-membership pair for the same element, which makes it more suitable for real-world decision-making. The expert analysis and multi-criteria problem as fuzzy and intuitionistic fuzzy themes had developed different shapes of fuzzy number such as triangular trapezoidal, pentagonal…. decagonal numbers. The multi numbers offer more accurate modeling for uncertain numerical data as their intuitionistic versions allow both multi membership and multi non-membership functions.

To compare two fuzzy membership function, the correlation measure was used by **Murthy, Pal & Majumder (1985) [7].** Later, in depth the correlation measure for the fuzzy sets was defined by **Chiang & Lin (1999) [4]** and discussed by **Chaudhuri & Bhattacharya (2001) [3]**. The concept was extended to intuitionistic fuzzy set by **Gerstenkorn & Manko (1991) [5]**. The correlation coefficient was introduced by **Mitchell (2004) [6]** and it was developed by **Wenyi Zeng & Hongxing Li (2007) [11]**.

It was found that the two intuitionistic fuzzy multi numbers were closely associated; when there was a high similarity nearer to one, while a value close to 0 shows low similarity. Likewise, while dealing with fuzzy and intuitionistic fuzzy numbers for comparison. the correlation measure can be used to find how closely two intuitionistic fuzzy multi numbers were related.

This article was focused on studying and proposing, the correlation measure for Intuitionistic Fuzzy Multi Numbers. The section 2, presents the basic definitions required for the study and the section 3, provides the definitions of Correlation Measure of IFMNs for all the categories namely, triangular trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, and decagonal forms. The section 4, explains the numerical evaluation, the worked-out examples which shows how the correlation measure are actually calculated and also authenticate our findings.

2. PRELIMINARIES

In the section, the basic definition and concepts were discussed in order to support the proposed study.

2.1 Definition:
Let X be a nonempty set. A **Fuzzy Set (FS)** A in X is given by A= $\{\langle x,\mu_A(x)\rangle / x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A (i.e.) $\mu_A(x) \in [0,1]$ is the membership of $x \in X$ in A.

2.2 Definition:
An **Intuitionistic Fuzzy Set (IFS)** A in X is given by A= $\{(x, \mu_A(x), \vartheta_A(x))/x\epsilon X\}$, where μ_A: X→ [0,1] , $\vartheta_A: X \to [0,1]$ with the condition $0 \le \mu_A(x) + \vartheta_A(x) \le 1, \forall x \in X$. Here, $\mu_A(x) and\ \vartheta_A(x) \in [0,1]$ denote the membership and the non-membership function of the fuzzy set A; for each intuitionistic fuzzy set in X, $\pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0$ for all x∈ X that is $\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ is the hesitancy degree of x ∈ X.
The complementary set A^c of A is defined as $A^c = \{(x, \vartheta_A(x), \mu_A(x)/x \in X)\}$

2.3 Definition:
Let X be a nonempty set. A **Fuzzy Multi Set (FMS)** A in X is characterized by the count membership function Mc such that Mc: X → Q where Q is the set of all crisp multi sets in [0,1]. Hence for any x∈ X, Mc(x) is crisp multi set from [0,1]. The membership sequence is defined as
$(\mu_A^1(x), \mu_A^2(x) \dots \dots \dots \mu_A^p(x))$ where$\mu_A^1(X) \ge \mu_A^2(x) \dots \dots \ge \mu_A^p(x)$.
Therefore, An FMS A is given by A = $\{(x, \mu_A^1(x), \mu_A^2(x), \dots \dots \dots \mu_A^p(x)) / x \in X\}$

2.4 Definition:
Let X be a nonempty set. An **Intuitionistic Fuzzy Multi Set (IFMS)** A in X is characterized by two functions namely count membership function Mc and count non-membership function NMc such that Mc: X→ Q where Q is the set of all crisp multi set in [0,1] whose membership sequence is defined as $(\mu_A^1(x), \mu_A^2(x), \dots \dots \dots \mu_A^p(x))$ where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$ and the corresponding non-membership sequence NMc (x) is defined as $(\vartheta_A^1(x), \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x))$ where the non-membership can be either decreasing or increasing function. Such that $0 \le \mu_A^I(X) + \vartheta_A^i(x) \le 1, \forall x \in X$ and
i = 1, 2, ……, p. Therefore, an IFMS A is given by
A= $\{((x, \mu_A^i(x), \mu_A^2(x), \dots \dots \mu_A^p(x), (\vartheta_A^1(x), \vartheta_A^2(x), \dots \dots \dots \vartheta_A^p(x)) / x \in X\}$
where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \mu_A^p(x)$

2.5 Definition:
Let A be the fuzzy set. Consider, A = [a, c, b] is the **Fuzzy Number (FN)** with a, b, c in R
A fuzzy set A on R is said to be fuzzy number if the following conditions are satisfied,
1. A is convex fuzzy set $\lambda x_1 + (1 - \lambda)x_2\ \epsilon \alpha_A$, $\lambda \epsilon [0,1]$
2. A is normalized fuzzy set H(A)=1
3. The supp(A) must be bounded

2.6 Definition:
An intuitionistic fuzzy subset A = { $(x, \mu_A(x), \vartheta_A(x))$ / $x\epsilon X$ } of the real line R is called an **Intuitionistic Fuzzy Number (IFN)** if there exists m in r such that $\mu_A(m)$ = 1 and $\vartheta_A(m)$ = 0, $\mu_A(x)$ is a continuous function from R→ [0,1] such that $0 \le \mu_A(x) + \vartheta_A(x) \le 1$ for all $x \in X$

2.7 Definition:
A **Fuzzy Multi Number (IFM)** is a generalization of a regular real number. It does not refer to a single value but rather to a connected set of possible values, where each possible value has its weight between 0 and 1. The weight is called the membership function. The membership sequence is in the form $(\mu_A^1(x),\ \mu_A^2(x) \dots \dots \mu_A^p(x))$
where $\mu_A^1(x) \ge \mu_A^2(x) \ge \dots \dots \dots \ge \mu_A^p(x))$

2.8 Definition:
An **Intuitionistic Fuzzy Multi Number (IFMN)** is a subset of Intuitionistic Fuzzy Set
A = { (x, $\mu_A(x), \vartheta_A(x)$) / $x\epsilon X$ } of the real line R, if there exists m in R such that $\mu_A(m) = 1$ and $\vartheta_A(m) = 0$ $\mu_A(x)$ is a continuous function from R [0,1] and $\vartheta_A(x)$ is a continuous function from R [0,1] such that $0 \le \mu_A(x) + \vartheta_A(x) \le 1$ for all $x \in X$The membership sequence is in the form { $(\mu_A^1(x),\ \mu_A^2(x) \dots \dots \mu_A^p(x)), ((\vartheta_A^1(x),\ \vartheta_A^2(x) \dots \dots \dots \dots \vartheta_A^p(x)))$}
where $(\mu_A^1(x) \ge \mu_A^2(x) \ge \dots \dots \dots \ge \mu_A^p(x))$ and $(\vartheta_A^1(x) \ge \vartheta_A^2(x) \ge \dots \dots \dots \ge \vartheta_A^p(x))$

2.9 Definition:
The **Cardinality** of the membership function Mc(x) is the length of an element x in the Intuitionistic Fuzzy Multi Number A denoted as η, defined as η = Mc(x)
If A, B and C are the FMS defined on X, then their cardinality η = Max {η(A), η(B),$\eta(C)$}

2.10 Definition:
$\rho(A,B)$ is said to be the **Similarity Measure** between A and B, where A, B ∈ X and X is an FS, as $\rho(A,B)$ satisfies the following properties
1.$\rho(A,B) \in [0,1]$
2.$\rho(A,B) = 1 \; if \; and \; only \; if \; A = B$
3.$\rho(A,B) = \rho(B,A)$

2.11 Definition:
Let A = $\{(x_i, \mu_A(x_i))/x_i \in X\}$ and B = $\{(x_i, \mu_B(x_i))/x_i \in X\}$ be two **FSs** on the finite universe of discourse X = $\{x_1, x_2, \dots \dots, x_n\}$,then the correlation coefficient of A and B is the **Fuzzy Correlation Measure** defined as
$\rho_{FS}(A,B) = \frac{C_{FS}(A,B)}{\sqrt{C_{FS}(A,A) * C_{FS}(B,B)}}$
where $C_{FS}(A,B) = \sum_{i=1}^{n} (\mu_A(x_i), \mu_B(x_i))$ $C_{FS}(A,A) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_A(x_i))$
$C_{FS}(B,B) = \sum_{i=1}^{n} (\mu_B(x_i)\mu_B(x_i))$

2.12 Definition:
Let X = $\{x_1, x_2, \dots \dots x_n\}$ be the finite universe of discourse and A = $\{(x_i, \mu_A(x), \vartheta_A(x))/x_i \in X\}$, B = $\{(x_i, \mu_B(x_i), \vartheta_B(x_i))/x_i \in X\}$ be two **IFSs** then the correlation coefficient of A and B is the **Intuitionistic Fuzzy Correlation measure** defined as $\rho_{IFS}(A,B) = \frac{C_{IFS}(A,B)}{\sqrt{C_{IFS}(A,A) * C_{IFS}(B,B)}}$
where $C_{IFS}(A,B) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_B(x_i) + \vartheta_A(x_i)\vartheta_B(x_i))$
$C_{IFS}(A,A) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_A(x_i) + \vartheta_A(x_i)\vartheta_A(x_i))$
$C_{IFS}(B,B) = \sum_{i=1}^{n} (\mu_B(x_i)\mu_B(x_i) + \vartheta_B(x_i)\vartheta_B(x_i))$

2.13 Definition:
Let X = $\{x_1, x_2, \dots \dots \dots, x_n\}$ be the finite universe of discourse and A = $\left\{\left(x_{i,}\mu_A^j(x_i)\right)/x_i \epsilon X\right\}$
B= $\left\{\left(x_{i,}\mu_A^j(x_i)\right)/x_i \epsilon X\right\}$ be the two FMSs then the correlation of A and B is the **Fuzzy Multi Correlation Measure** defined as ρ_{FMS}(A, B) = $\frac{C_{FMS}(A,B)}{\sqrt{C_{FMS}(A,A) \times C_{FMS}(B,B)}}$
where $C_{FMS}(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_A^j(x_i)\; \mu_B^j(x_i)\right)\right\}$ $C_{FMS}(A,A) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_A^j(x_i)\; \mu_A^j(x_i)\right)\right\}$
$C_{FMS}(B,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_B^j(x_i)\; \mu_B^j(x_i)\right)\right\}$

2.14 Definition:
Let X = $\{x_1, x_2, \dots \dots \dots, x_n\}$ be the finite universe of discourse and A = $\left\{\left(x_{i,}\mu_A^j(x_i), \vartheta_A^j(x_i)\right)/x_i \epsilon X\right\}$
B= $\left\{\left(x_{i,}\mu_A^j(x_i), \vartheta_A^j(x_i)\right)/x_i \epsilon X\right\}$ be the **IFMSs** consisting of the membership and non-membership function, then the correlation of A and B is the **Intuitionistic Fuzzy Multi Correlation Measure** defined as
ρ_{IFMS}(A, B) = $\frac{C_{IFMS}(A,B)}{\sqrt{C_{IFMS}(A,A) \times C_{IFMS}(B,B)}}$
$C_{IFMS}(A,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\{\sum_{i=1}^{n}(\mu_A^j(x_i)\; \mu_B^j(x_i) + \vartheta_A^j(x_i)\; \vartheta_B^j(x_i))\}$
$C_{IFMS}(A,A) = \frac{1}{\eta}\sum_{j=1}^{\eta}\{\sum_{i=1}^{n}(\mu_A^j(x_i)\; \mu_A^j(x_i) + \vartheta_A^j(x_i)\; \vartheta_A^j(x_i))\}$
$C_{IFMS}(B,B) = \frac{1}{\eta}\sum_{j=1}^{\eta}\{\sum_{i=1}^{n}(\mu_B^j(x_i)\; \mu_B^j(x_i) + \vartheta_B^j(x_i)\; \vartheta_B^j(x_i))\}$

3. Correlation Measure for the Intuitionistic Fuzzy Multi Numbers

In this section we introduced a correlation measure for intuitionistic fuzzy multi numbers like triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, and decagonal. And also discussed the definition of correlation measure, formula for IFMNs and checked the properties with numerical examples for the properties.

3.1 Definition:
Let A and B be two **IFMNs** on the finite universe of discourse X = $\{x_1, x_2, \dots \dots, x_n\}$,then the correlation coefficient of A and B is the **Fuzzy Correlation Measure** defined as

$$\rho_{IFMN}(A,B) = \frac{C_{IFMN}(A,B)}{\sqrt{C_{IFMN}(A,A) * C_{IFMN}(B,B)}} \quad \text{-----------} \; (3.1)$$

This formula can be presented in various type of **IFMNs** like triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal and decagonal which are derived below

3.2 Definition:
Let X = {x_1, x_2, ………., x} be the finite universe of discourse and A = $\{(a_1^i, b_1^i, c_1^i)(d_1^i, b_1^i, e_i^i)\}$
B = $\{(a_2^i, b_2^i, c_2^i)(d_2^i, b_2^i, e_2^i)\}$ then the correlation coefficient of A and B, the **triangular IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + 2(b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e_1^i \times e_2^i)]$$
$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + 2(b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e_1^i \times e_1^i)]$$
$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + 2(b_2^i \times b_2^i) + (c_2^i \times c_2^i) + (d_2^i \times d_2^i) + (e_2^i \times e_2^i)]$$

3.3 Definition:
Let X = $\{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and
A =$\{(a_1^i, b_1^i, c_1^i, d_1^i)(e_1^i, b_1^i, c_1^i, f_1^i)\}$ B = $\{(a_2^i, b_2^i, c_2^i, d_2^i,)(e_2^i, b_2^i, c_2^i, f_2^i)\}$ then the correlation coefficient of A and B , the **trapezoidal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^i) + 2(b_1^i \times b_2^i) + 2(c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e_1^i \times e_2^i) + (f_1^i \times f_2^i)]$$
$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + 2(b_1^i \times b_1^i) + 2(c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e_1^i \times e_1^i) + (f_1^i \times f_1^i)]$$
$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + 2(b_2^i \times b_2^i) + 2(c_2^i \times c_2^i) + (d_2^i \times d_2^i) + (e_2^i \times e_2^i) + (f_2^i \times f_2^i)]$$

3.4 Definition:
Let X = $\{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and
A =$\{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i)(f_1^i, g_1^i, c_1^i, h_1^i, i_1^i)\}$ B = $\{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i)(f_2^i, g_2^i, c_2^i, h_2^i, i_2^i)\}$ then the correlation coefficient of A and B , the **pentagonal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + (b_1^i \times b_2^i) + 2(c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e_1^i \times e_2^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i)]$$
$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + 2(c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e_1^i \times e_1^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i)]$$
$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + 2(c_2^i \times c_2^i) + (d_2^i \times d_2^i) + (e_2^i \times e_2^i) + (f_2^i \times f_2^i) + (g_2^i \times h_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i)]$$

3.5 Definition:
Let X = $\{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and
A=$\{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i)(g_1^i, h_1^i, c_1^i, d_1^i, i_1^i, j_1^i)\}$ B = $\{(a_2^i, b_2^i, c_2^i, d_2^i, e_1^i, f_1^i)(g_2^i, h_2^i, c_2^i, d_2^i, i_2^i, j_2^i)\}$ then the correlation coefficient of A and B, the **hexagonal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + (b_1^i \times b_2^i) + 2(c_1^i \times c_2^i) + 2(d_1^i \times d_2^i) + (e_1^i \times e_2^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i) + (j_1^i \times j_2^i)]$$
$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + 2(c_1^i \times c_1^i) + 2(d_1^i \times d_1^i) + (e_1^i \times e_1^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i) + (j_1^i \times j_1^i)]$$
$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + 2(c_2^i \times c_2^i) + 2(d_2^i \times d_2^i) + (e_2^i \times e_2^i) + (f_2^i \times f_2^i) + (g_2^i \times h_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i) + (j_2^i \times j_2^i)]$$

3.6 Definition:
Let X = $\{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and A=$\{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i)(h_1^i, i_1^i, j_1^i, d_1^i, k_1^i, l_1^i)\}$
B = $\{(a_2^i, b_2^i, c_2^i, d_2^i, e_1^i, f_1^i, g_2^i)(h_2^i, i_2^i, j_2^i, d_2^i, k_2^i, l_2^i)\}$ then the correlation coefficient of A and B, the **heptagonal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + 2(d_1^i \times d_2^i) + (e_1^i \times e_2^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i) + (j_1^i \times j_2^i) + (k_1^i \times k_2^i) + (l_1^i \times l_2^i)]$$
$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + 2(d_1^i \times d_1^i) + (e_1^i \times e_1^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i) + (j_1^i \times j_1^i) + (k_1^i \times k_1^i) + (l_1^i \times l_1^i)]$$
$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + 2(d_2^i \times d_2^i) + (e_2^i \times e_2^i) + (f_2^i \times f_2^i) + (g_2^i \times h_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i) + (j_2^i \times j_2^i) + (k_2^i \times k_2^i) + (l_2^i \times l_2^i)]$$

3.7 Definition:
Let X $= \{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and
A=$\{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i, h_1^i)(i_1^i, j_1^i, k_1^i, d_1^i, e_1^i, l_1^i, m_1^i, n_1^i)\}$
B = $\{(a_2^i, b_2^i, c_2^i, d_2^i, e_1^i, f_1^i, g_2^i, h_2^i)(i_2^i, j_2^i, k_2^i, d_2^i, e_2^i, l_2^i, m_2^i, n_2^i)\}$ then the correlation coefficient of A and B, the **octagonal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + 2(d_1^i \times d_2^i) + 2(e_1^i \times e_2^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i) + (j_1^i \times j_2^i) + (k_1^i \times k_2^i) + (l_1^i \times l_2^i) + (m_1^i \times m_2^i) + (n_1^i \times n_2^i)]$$

$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + 2(d_1^i \times d_1^i) + 2(e_1^i \times e_1^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i) + (j_1^i \times j_1^i) + (k_1^i \times k_1^i) + (l_1^i \times l_1^i) + (m_1^i \times m_1^i) + (n_1^i \times n_1^i)]$$

$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + 2(d_2^i \times d_2^i) + 2(e_2^i \times e_2^i) + (f_2^i \times f_2^i) + (g_2^i \times g_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i) + (j_2^i \times j_2^i) + (k_2^i \times k_2^i) + (l_2^i \times l_2^i) + (m_2^i \times m_2^i) + (n_2^i \times n_2^i)]$$

3.8 Definition:
Let X $= \{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and
A=$\{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i, h_1^i, i_1^i)(j_1^i, k_1^i, l_1^i, m_1^i, e_1^i, n_1^i, o_1^i, p_1^i, q_1^i)\}$
B = $\{(a_2^i, b_2^i, c_2^i, d_2^i, e_1^i, f_1^i, g_2^i, h_2^i, i_1^i)(j_2^i, k_2^i, l_2^i, m_2^i, e_2^i, n_2^i, o_2^i, p_2^i, q_2^i)\}$ then the correlation coefficient of A and B, the **nonagonal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + 2(e_1^i \times e_2^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i) + (j_1^i \times j_2^i) + (k_1^i \times k_2^i) + (l_1^i \times l_2^i) + (m_1^i \times m_2^i) + (n_1^i \times n_2^i) + (o_1^i \times o_2^i) + (p_1^i \times p_2^i) + (q_1^i \times q_2^i)]$$

$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + 2(e_1^i \times e_1^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i) + (j_1^i \times j_1^i) + (k_1^i \times k_1^i) + (l_1^i \times l_1^i) + (m_1^i \times m_1^i) + (n_1^i \times n_1^i) + (o_1^i \times o_1^i) + (p_1^i \times p_1^i) + (q_1^i \times q_1^i)]$$

$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + 2(d_2^i \times d_2^i) + 2(e_2^i \times e_2^i) + (f_2^i \times f_2^i) + (g_2^i \times h_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i) + (j_2^i \times j_2^i) + (k_2^i \times k_2^i) + (l_2^i \times l_2^i) + (m_2^i \times m_2^i) + (n_2^i \times n_2^i) + (o_2^i \times o_2^i) + (p_2^i \times p_2^i) + (q_2^i \times q_2^i)]$$

3.9 Definition:
Let X $= \{x_1, x_2, \dots \dots \dots x_n\}$ be the finite universe of discourse and
A=$\{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i, h_1^i, i_1^i, j_1^i)(k_1^i, l_1^i, m_1^i, n_1^i, e_1^i, f_1^i, o_1^i, p_1^i, q_1^i, r_1^i)\}$
B = $\{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i, f_1^i, g_2^i, h_2^i, i_1^i, j_2^i)(k_2^i, l_2^i, m_2^i, n_2^i, e_2^i, f_2^i, o_2^i, p_2^i, q_2^i, r_2^i)\}$ then the correlation coefficient of A and B, the **decagonal IFMNs** expressed in equation 3.1 was considered where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_2^1) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + 2(e_1^i \times e_2^i) + 2(f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i) + (j_1^i \times j_2^i) + (k_1^i \times k_2^i) + (l_1^i \times l_2^i) + (m_1^i \times m_2^i) + (n_1^i \times n_2^i) + (o_1^i \times o_2^i) + (p_1^i \times p_2^i) + (q_1^i \times q_2^i) + (r_1^i \times r_2^i)]$$

$$C_{IFMN}(A,A) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + 2(e_1^i \times e_1^i) + 2(f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i) + (j_1^i \times j_1^i) + (k_1^i \times k_1^i) + (l_1^i \times l_1^i) + (m_1^i \times m_1^i) + (n_1^i \times n_1^i) + (o_1^i \times o_1^i) + (p_1^i \times p_1^i) + (q_1^i \times q_1^i) + (r_1^i \times r_1^i)]$$

$$C_{IFMN}(B,B) = \frac{1}{\eta}\sum_{i=1}^{\eta}[(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + 2(d_2^i \times d_2^i) + 2(e_2^i \times e_2^i) + 2(f_2^i \times f_2^i) + (g_2^i \times h_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i) + (j_2^i \times j_2^i) + (k_2^i \times k_2^i) + (l_2^i \times l_2^i) + (m_2^i \times m_2^i) + (n_2^i \times n_2^i) + (o_2^i \times o_2^i) + (p_2^i \times p_2^i) + (q_2^i \times q_2^i) + (r_2^i \times r_2^i)]$$

3.10 Proposition
The proposed Correlation Measure ρ_{IFMN}(A, B) between IFMNs A and B satisfies the following properties
1. 0 ≤ ρ_{IFMN} (A, B) ≤ 1
2. ρ_{IFMN}(A, B) =1 if and only if A = B
3. ρ_{IFMN} (A, B) = ρ_{IFMN} (B, A)

Proof
We have considered the triangular IFMNs of A and B for the verification process and the same can be generalized for the other categories trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, decagonal

1. 0 ≤ ρ_{IFMN} (A, B) ≤ 1 as the membership and the non-membership function of the IFMNs lies between 0 and 1, $\rho_{IFMN}(A, B)$ also lies between 0 and 1.

3.10.1 Example

Let $A = \begin{Bmatrix} (3,6,4)(2,6,5) \\ (4,7,3)(3,7,4) \\ (5,8,2)(4,8,3) \end{Bmatrix}$ $B = \begin{Bmatrix} (7,1,3)(6,1,5) \\ (5,4,2)(3,4,8) \\ (5,8,3)(4,8,6) \end{Bmatrix}$

here the cardinality $\eta = 3$ C_{IFMN}(A, B) = 132.66 C_{IFMN}(A, A) = 162 C_{IFMN}(B, B) = 156.33then $\rho_{IFMN}(A,B) = \frac{132.66}{\sqrt{162\times156.33}} = 0.8336$ hence $0 \le \rho_{IFMN}$ (A, B) ≤ 1

2. ρ_{IFMN}(A, B) =1 if and only if A = B

- Let two IFMN a and b be equal (i.e.) A = B

  $C_{IFMN}(A,A) = C_{IFMN}(B,B) = \frac{1}{\eta}\{\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + 2(b^i \times b^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e_1^i \times e_1^i)]\}$

  $C_{IFMN}(A,B) = \frac{1}{\eta}\{\sum_{i=0}^{\eta}[(a_1^i \times a_2^i) + 2(b^i \times b^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e_1^i \times e_2^i)]\} = \frac{1}{\eta}\{\sum_{i=1}^{\eta}[(a_1^i \times a_1^i) + 2(b^i \times b^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e_1^i \times e_1^i)]\} = C_{IFMN}(A,A)$

  Hence $\rho_{IFMN}(A,B) = \frac{C_{IFMN}(A,B)}{\sqrt{C_{IFMN}(A,A)\times C_{IFMN}(B,B)}} = \frac{C_{IFMN}(A,B)}{\sqrt{C_{IFMN}(A,A)\times C_{IFMN}(B,B)}} = 1$

- Let the $\rho_{IFMN}(A,B) = 1$

  The unit measure possible only if $\rho_{IFMN}(A,B) = \frac{C_{IFMN}(A,B)}{\sqrt{C_{IFMN}(A,A)\times C_{IFMN}(B,B)}} = 1$

  This refers that $C_{IFMN}(A,B) = C_{IFMN}(A,) = C_{IFMN}(B,B)$

  hence A= B.

3.10.2 Example

Let $A = \begin{Bmatrix} (2,4,5)(1,4,6) \\ (3,5,6)(2,5,8) \\ (2,10,7)(2,10,9) \end{Bmatrix}$ $B = \begin{Bmatrix} (2,4,5)(1,4,6) \\ (3,5,6)(2,5,8) \\ (2,10,7)(2,10,9) \end{Bmatrix}$

here the cardinality $\eta = 3$ C_{IFMN}(A, B) =199.66 C_{IFMN}(A, A) = 199.66 C_{IFMN}(B, B) = 199.66

then $\rho_{IFMN}(A,B) = \frac{199.66}{\sqrt{199.66\times199.66}} = 1$ hence ρ_{IFMN}(A, B) =1 if and only if A = B

3. $\rho_{IFMN}(A,B) = \rho_{IFMN}(B,A)$

It is obvious that

$$\rho_{IFMN}(A,B) = \frac{C_{IFMN}(A,B)}{\sqrt{C_{IFMN}(A,A)\times C_{IFMN}(B,B)}} = \frac{C_{IFMN}(B,A)}{\sqrt{C_{IFMN}(A,A)\times C_{IFMN}(B,B)}} = \rho_{IFMN}(B,A)$$

where

$$C_{IFMN}(A,B) = \frac{1}{\eta}\{\sum_{i=1}^{\eta}[(a_1^i \times a_2^i) + 2(b^i \times b^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e_1^i \times e_2^i)]\}$$

$$= \frac{1}{\eta}\{\sum_{i=1}^{\eta}[(a_2^i \times a_1^i) + 2(b^i \times b^i) + (c_2^i \times c_1^i) + (d_2^i \times d_1^i) + (e_2^i \times e_1^i)]\} = C_{IFMN}(B,A)$$

3.10.3 Example

Let $A = \begin{Bmatrix} (4,8,3)(3,8,4) \\ (5,7,2)(4,8,3) \\ (6,3,1)(5,3,2) \end{Bmatrix}$ $B = \begin{Bmatrix} (9,8,7)(4,8,9) \\ (6,5,4)(3,5,6) \\ (3,2,1)(1,2,4) \end{Bmatrix}$

here the cardinality $\eta = 3$ C_{IFMN}(A, B) = 140 C_{IFMN}(A, A) = 143 C_{IFMN}(B, B) = 180.33

then $\rho_{IFMN}(A,B) = \frac{140}{\sqrt{143\times180.33}}$ = 0.8718 similarly $\rho_{IFMN}(B,A) = \frac{140}{\sqrt{180.33\times143}}$ = 0.8718

Hence, $\rho_{IFMN}(A,B) = \rho_{IFMN}(B,A)$

4. NUMERICAL EVALUATION

In the section, the correlation measure for different Intuitionistic Fuzzy Multi Numbers (IFMN) were analyzed by exercising several examples considering both in equal cardinalities (table 4.1 & 4.2) and unequal cardinalities (table 4.3 & 4.4). These examples include the categories of triangular trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, decagonal IFMNs. These numerical values are computed to show how the correlation measure can be considered for any real-time multi decision application as the results of any categories lies from 0 to 1.

(a) Firstly, the equal cardinality **of $\eta = 3$** was considered for the two IFMNs A and B.

For all the categories of triangular trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, decagonal IFMNs, the same three data of intuitionistic fuzzy numbers were considered and represented in the following table.

Table 4.1: Numerical examples of equal cardinality

Type of the IFMNs	**Numerical Examples: $\eta = 3$ was considered**
Triangular	$A = \left\{\begin{matrix}(2,4,5)(1,4,6)\\(3,5,6)(2,5,8)\\(2,10,7)(2,10,9)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(3,5,7)(2,5,9)\\(3,2,8)(3,2,10)\\(4,7,9)(2,7,11)\end{matrix}\right\}$
Trapezoidal	$A = \left\{\begin{matrix}(2,3,5,6)(0,3,5,7)\\(1,2,4,5)(1,2,4,9)\\(3,5,6,8)(2,5,6,7)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(4,5,9,11)(2,5,9,14)\\(6,11,13,15)(4,11,13,15)\\(2,3,6,8)(1,3,6,14)\end{matrix}\right\}$
Pentagonal	$A = \left\{\begin{matrix}(1,3,5,7,10)(0,3,5,6,11)\\(2,2,4,6,9)(2,4,4,7,13)\\(3,5,6,8,12)(1,2,6,8,16)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(2,4,8,13,15)(1,3,8,12,17)\\(0,3,7,11,17)(0,4,7,11,19)\\(4,5,8,10,14)(0,5,8,13,16)\end{matrix}\right\}$
Hexagonal	$A = \left\{\begin{matrix}(1,3,5,6,8,11)(1,5,5,6,11,12)\\(0,3,5,7,9,13)(0,4,5,7,13,14)\\(2,3,7,9,12,13)(1,5,7,9,11,15)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(6,7,8,9,11,13)(2,3,8,9,9,16)\\(5,6,8,10,12,14)(4,5,8,10,12,14)\\(2,3,5,9,11,15)(0,3,5,9,10,17)\end{matrix}\right\}$
Heptagonal	$A = \left\{\begin{matrix}(1,2,3,4,5,6,7)(0,4,2,4,7,8,10)\\(2,3,7,9,12,13,10)(2,5,7,9,11,12,13)\\(2,5,7,11,4,8,9)(1,4,8,11,8,9,10)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(7,8,9,10,11,12,13)(6,7,19,10,7,7,14)\\(1,2,3,6,8,9,9)(0,5,6,6,8,9,10)\\(0,4,8,9,13,16,7)(0,5,6,9,12,13,10)\end{matrix}\right\}$
Octagonal	$A = \left\{\begin{matrix}(7,8,9,10,11,12,13,14)(3,6,7,10,11,11,12,15)\\(6,7,8,9,11,13,14,15)(0,2,3,9,11,12,13,16)\\(5,7,9,10,12,14,15,16)(2,4,5,10,12,13,15,17)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(2,4,6,7,8,9,10,11)(1,2,3,7,8,8,9,14)\\(1,2,3,4,6,8,11,14)(1,3,4,4,6,9,11,15)\\(0,3,4,7,8,12,13,15)(0,5,7,7,8,14,15,16)\end{matrix}\right\}$
Nonagonal	$A = \left\{\begin{matrix}(6,8,10,6,8,11,7,9,10)(5,6,9,5,8,13,5,7,12)\\(5,7,9,3,7,8,7,18,12)(4,7,8,4,7,3,2,19,11)\\(7,8,9,6,8,9,8,11,12)(2,3,4,4,8,7,9,10,13)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(2,3,6,8,3,1,3,6,14)(1,6,2,9,3,4,6,9,15)\\(4,7,9,13,2,3,7,9,13)(3,1,5,7,2,4,6,9,19)\\(1,3,5,7,1,0,3,5,6)(1,5,6,8,1,1,2,6,8)\end{matrix}\right\}$
Decagonal	$A = \left\{\begin{matrix}(1,2,3,4,5,6,7,8,9,10)(0,9,8,7,5,6,4,3,2,19)\\(2,4,5,3,4,6,2,4,7,6)(1,3,5,7,4,6,5,9,8,10)\\(1,3,5,2,3,6,1,3,7,8)(1,2,5,8,3,6,3,5,12,16)\end{matrix}\right\}$ $B = \left\{\begin{matrix}(5,9,10,8,9,11,7,18,12,13)(4,5,9,10,9,11,7,9,12,14)\\(1,3,5,1,4,6,3,5,9,12)(0,2,6,8,4,6,5,8,3,15)\\(7,8,9,10,11,12,13,14,15,16)(6,7,8,9,10,11,13,16,17,18)\end{matrix}\right\}$

By considering the proposed new correlation measure of IFMNs, the resultant values are tabulated below to analyze in depth for all the categories of IFMNs.

Table 4.2: Numerical evaluation of equal cardinality

Type of the IFMNs	**Correlation Measure of IFMNs**
Triangular	$C_{IFMN}(A, B) = 204.66$ $C_{IFMN}(A, A) = 199.66$ and $C_{IFMN}(B, B) = 234.33$ then $\rho_{IFMN} = \frac{204.66}{\sqrt{199.66 \times 234.33}}$ = **0.94618**
Trapezoidal	$C_{IFMN}(A, B) = 320$ $C_{IFMN}(A, A) = 195$ and $C_{IFMN}(B, B) = 662$

	then $\rho_{IFMN} = \frac{320}{\sqrt{195\times662}}$ = **0.89064**
Pentagonal	C_{IFMN}(A, B) = 658.33 C_{IFMN}(A, A) = 467.66 and C_{IFMN}(B, B) = 971.66 then $\rho_{IFMN} = \frac{658.33}{\sqrt{467.66\times971.66}}$ =**0.97662**
Hexagonal	C_{IFMN}(A, B) = 844.66 C_{IFMN}(A, A) = 783.66 and C_{IFMN}(B, B) = 971.66 then $\rho_{IFMN} = \frac{658.33}{\sqrt{467.66\times1031.33}}$ = **0.93954**
Heptagonal	C_{IFMN}(A, B) = 796.66 C_{IFMN}(A, A) = 781.66 and C_{IFMN}(B, B) = 1117.66 then $\rho_{IFMN} = \frac{796.66}{\sqrt{781.66\times1117.66}}$ = **0.85233**
Octagonal	C_{IFMN}(A, B) = 1410.33 C_{IFMN}(A, A) = 1834 and C_{IFMN}(B, B) = 1443.66 then $\rho_{IFMN} = \frac{1410.33}{\sqrt{1834\times1443.66}}$ = **0.86674**
Nonagonal	C_{IFMN}(A, B) = 858.33 C_{IFMN}(A, A) = 1307.66 and C_{IFMN}(B, B) = 844 then $\rho_{IFMN} = \frac{1858.33}{\sqrt{1307.66\times844}}$ = **0.81703**
Decagonal	C_{IFMN}(A, B) = 1086 C_{IFMN}(A, A) = 809 and C_{IFMN}(B, B) =1931.33 then $\rho_{IFMN} = \frac{1086}{\sqrt{809\times1931.33}}$ = **0.869**

(b) Secondly, the unequal cardinality IFMNs example were considered. Here the IFMNs A and B do not contain the same number of components. For all the categories of triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, decagonal IFMNs; the different cardinality of A and B were considered and represented in the following table 4.3 & 4.4.

Table 4.3: Numerical examples of unequal cardinality

IFMNs	**Numerical examples of unequal cardinality**
Triangular	A = $\begin{Bmatrix}(2,3,5)(1,3,6)\\(5,2,1)(4,2,1)\\(7,4,3)(6,4,5)\end{Bmatrix}$ $B = \{(3,5,6)(2,5,8)\}$ Here, $\eta\ of\ A = 3$ and $\eta\ of\ B = 1$
Trapezoidal	A = $\begin{Bmatrix}(2,3,5,6)(0,3,5,7)\\(1,2,4,5)(1,2,4,9)\\(8,5,6,3)(2,5,6,9)\\(5,4,3,8)(3,4,3,9)\end{Bmatrix}$ B = {(4,5,9,11)(2,5,9,14)} Here, $\eta\ of\ A = 4$ and $\eta\ of\ B = 1$
Pentagonal	A = {(4,7,9,12,15)(3,6,9,11,8)} B = $\begin{Bmatrix}(1,3,5,7,10)(0,3,5,6,11)\\(2,2,4,6,9)(2,4,4,7,13)\\(3,5,6,8,12)(1,2,6,8,16)\end{Bmatrix}$ Here, $\eta\ of\ A = 1$ and $\eta\ of\ B = 3$
Hexagonal	A = {(1,2,3,4,5,6)(1,3,3,4,9,14)} B = $\begin{Bmatrix}(1,3,5,6,8,11)(1,5,5,6,11,12)\\(0,3,5,7,9,13)(0,4,5,7,13,14)\\(2,3,7,9,12,13)(1,5,7,9,11,15)\end{Bmatrix}$ Here, $\eta\ of\ A = 1$ and $\eta\ of\ B = 3$
Heptagonal	A = $\begin{Bmatrix}(2,3,5,8,6,7,4)(1,5,6,8,9,7,5)\\(5,3,5,6,8,9,7)(4,8,7,6,7,8,9)\\(7,3,4,5,6,7,5)(4,5,6,5,5,67)\\(5,4,3,2,9,8,7)(4,3,5,2,7,8,9)\end{Bmatrix}$ B = {(5,4,3,1,4,5,6)(4,8,7,1,5,4,8)} Here, $\eta\ of\ A = 4$ and $\eta\ of B = 1$
Octagonal	A=$\begin{Bmatrix}(4,8,9,4,5,4,8,7)(3,4,3,4,5,6,8,9)\\(8,4,5,3,2,8,4,10)(7,8,9,3,2,7,5,15)\\(2,3,4,5,6,7,8,9)(1,2,3,5,6,8,9,10)\end{Bmatrix}$ B={(5,4,8,9,4,9,6,7)(4,3,9,9,4,10,7,8)} Here, $\eta\ of\ A = 3$ and $\eta\ of\ B = 1$
Nonagonal	A ={(5,4,3,5,3,8,4,5,3)(4,8,7,5,3,7,4,8,5)} B =$\begin{Bmatrix}(4,2,8,9,5,4,3,5,10)(3,9,8,7,5,13,12,10,12)\\(5,4,8,4,2,9,4,3,8)(2,8,4,5,2,13,12,18,16)\\(4,5,4,3,1,8,7,5,9)(3,5,13,10,1,13,14,15,18)\end{Bmatrix}$ Here, $\eta\ of\ A = 1$ and $\eta\ of\ B = 3$
Decagonal	A =$\begin{Bmatrix}(8,4,3,2,1,8,5,4,3,2)(4,3,8,7,1,8,4,5,8,3)\\(4,8,7,5,2,1,8,7,5,3)(3,9,4,3,2,1,5,4,8,5)\\(5,8,4,3,5,4,9,4,7,4)(3,9,3,2,5,4,8,5,6,5)\end{Bmatrix}$ B = {(9,4,8,9,5,3,5,9,8,1)(8,9,0,3,5,3,8,5,8,8)}

	Here, $\eta\ of\ A = 3$ and $\eta\ of\ B = 1$

By considering the proposed new correlation measure of IFMNs, the resultant values are tabulated below to analyze in depth for all the categories of IFMNs.

Table 4.4: Numerical evaluation of unequal cardinality

Type of the IFMNs	Correlation Measure
Triangular	C_{IFMN}(A, B) = 101.33 C_{IFMN}(A, A) =95.33 and C_{IFMN}(B, B) = 163 then $\rho_{IFMN} = \frac{101.33}{\sqrt{95.33\times163}}$ = **0.81291**
Trapezoidal	C_{IFMN}(A, B) = 314.5 C_{IFMN}(A, A) = 203.5 and C_{IFMN}(B, B) = 549 then $\rho_{IFMN} = \frac{314.5}{\sqrt{203.5\times549}}$ = **0.9409**
Pentagonal	C_{IFMN}(A, B) = 698.33 C_{IFMN}(A, A) = 1086 and C_{IFMN}(B, B) = 469.67 then $\rho_{IFMN} = \frac{698.33}{\sqrt{1086\times469.67}}$ = **0.9778**
Hexagonal	C_{IFMN}(A, B) = 533 C_{IFMN}(A, A) = 403 and C_{IFMN}(B, B) = 784.67 then $\rho_{IFMN} = \frac{533}{\sqrt{403\times784.67}}$ = **0.948**
Heptagonal	C_{IFMN}(A, B) = 384.75 C_{IFMN}(A, A) =526.75 and C_{IFMN}(B, B) = 363 then $\rho_{IFMN} = \frac{(384.75)}{\sqrt{526.75\times363}}$ = **0.8799**
Octagonal	C_{IFMN}(A, B) = 644 C_{IFMN}(A, A) = 661 and C_{IFMN}(B, B) =659 then $\rho_{IFMN} = \frac{644}{\sqrt{661\times659}}$ = **0.9758**
Nonagonal	C_{IFMN}(A, B) = 719.667 C_{IFMN}(A, A) = 515 and C_{IFMN}(B, B) =1310 then $\rho_{IFMN} = \frac{719.667}{\sqrt{515\times1310}}$ = **0.8762**
Decagonal	C_{IFMN}(A, B) = 585 C_{IFMN}(A, A) = 566.33 and C_{IFMN}(B, B) =836 then $\rho_{IFMN} = \frac{585}{\sqrt{566.33\times836}}$ = **0.8502**

From the above tables 4.2 & 4.4, it was clear that the correlation measure values for both equal and unequal cardinality of the IFMNs for all the categories (triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, and decagonal) lies in between 0 and 1 and hence it was authenticated that the proposed measure was a well-defined one.

Conclusion

The basic concept required to understand the Correlation Measure for Intuitionistic Fuzzy Multi Number (IFMN) is discussed and proposed in this article. As, the Correlation Measure plays an important role in comparing and determining the deviation between any two data; the same concept was extended to the comparison of two Intuitionistic Fuzzy Multi Numbers. Here, it helps to analyze how closely the two IFMNs were related. The numerical evaluation shows the efficiency of the defined measure. Hence, this idea can be used in human decision making, patten recognitions and many practical applications.

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