SIMILARITY MEASURE BASED ON CORRELATION FOR FUZZY MULTI NUMBERS

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Abstract: The Correlation Measure is an important recent research topic as it has great practical potential in a various area, like decision making, medical diagnosis, machine learning, image processing, pattern recognition, etc. In this paper, the Correlation Measure of Fuzzy Multi Numbers (FMNs) is proposed for triangle, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, and decagonal. This Correlation concept of Fuzzy Multi Numbers is the extension of Intuitionistic Fuzzy Multi Set's Correlation Measure. Here, the properties of Correlation Measure were checked and some numerical examples were presented to authenticate the efficiency of the proposed measure.

Keywords: Similarity Measure, Correlation Measure, Fuzzy Sets, Fuzzy Numbers, Fuzzy Multi Sets, Fuzzy Multi Numbers.

1. INTRODUCTION

The Fuzzy Set *(FS)* was proposed by **Lotfi A. Zadeh [11]** in **1965** and it allows the uncertainty belong to a set with a membership degree (μ) between 0 and 1. As a Fuzzy set is widely used in various field like Pattern Recognition, medical diagnosis, logic programming, decision making, market prediction, etc. Several extensions have been proposed in recent decades to enhance the fuzzy theory's capacity for language to deal with uncertainty and incomplete information. Entropy and similarity measures are both important concepts in this real and are used significantly in fuzzy reasoning, pattern recognition, and clustering. Entropy measures the amount of fuzziness in a set defined by **Zadeh** in **1965 [1]**and similarity measures, the extent to which fuzzy sets are similar was defined by **Wang** in **1982 [8].**

The Fuzzy Number was proposed by **Dijkman et al [4]**, in which the quantity value is uncertain rather than exact as in the single valued numbers, and its membership function is monotonic on both sides of the largest membership values. Several authors like **Murthy and Pal [6]** investigated the Correlation between two Fuzzy Membership functions.

Chiang and Lin [2] studied the Correlation of Fuzzy sets and **Chaudhuri and Bhattacharya [1]** discussed the correlation between two fuzzy sets on same universal discourse. To address this fuzzy correlation measures for generalizations, the Pearson's correlation was developed by **Yu, 1993 and Liu, 2002 [10,5]**. **Rajarajeshwari and Uma [7]** studied the Correlation measure of Intuitionistic Fuzzy Multi sets (*IFMS*). Regarding decision-making under uncertainty, more recent studies have been created by **Hai et al., 2021 [3]** in correlation based and similarity-based methods**.**

To maintain repetitive information and to help accuracy in multi-criteria decision-making and medical diagnosis, **Yager (1986) [9]** put forth the idea of Fuzzy Multisets (*FMS*), because elements can have several degrees of membership. Since real world data is often ambiguous or multi valued classical correlation analysis is inadequate.

In this paper, the eight different Correlation Measure of *FMNs* for triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, Nanogonal and decagonal categories have been introduced also verified its proposition in section 3. Moreover, some numerical evaluations were illustrated in Section 4. In future, we will discuss the applications of the proposed Correlation Measure of *FMNs*.

2. PRELIMINARIES

In this section, we have discussed the basic concepts and definitions which are useful for the proposed study.

2.1 Definition

Let X be a nonempty set. A ***Fuzzy Set*** *(FS) A* in *X* is given by $A=\{\langle x, \mu_A(x)\rangle / x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set *A* (i.e.) $\mu_A(x) \in [0,1]$ is the membership of $x \in X$ in *A.*

2.2 Definition

A ***Multi Set*** (*MS*) is an unordered collection of objects in which, unlike an ordinary set, objects are allowed to repeat. Each individual occurrence of an object is a multi-set which is called its element.

2.3 Definition

Let X be a nonempty set. A ***Fuzzy Multi Set*** *(FMS)* A in X is characterized by the count membership function Mc such that Mc: X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1]. The membership sequence is defined as $\left(\mu_A^1(x), \mu_A^2(x), \dots \dots \dots \mu_A^p(x)\right)$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq, \dots \geq \mu_A^p(x)$.
Therefore, A *FMS A* is given by $A = \left\{\langle x, \left(\mu_A^1(x), \mu_A^2(x), \dots \dots \dots \mu_A^p(x)\right)\rangle / x \in X\right\}$.

2.4 Definition

The ***Cardinality*** of the membership function Mc(x) is the length of an element x in an *FMS* A denoted as η, defined as η = $|M_C$(x) |. If A, B and C are the *FMS* defined on *X,* then their cardinality η = Max {η(A), η(B), η(C)}.

2.5 Definition

A ***Fuzzy Number*** *(FN)* $\bar{A}$ is a convex normalized fuzzy set on the real line R such that there exist, at least one x ∈ R with $\mu_{\bar{A}}(x)$ = 1 and $\mu_{\bar{A}}(x)$ is piecewise continuous**.**

2.6 Definition

If X be a non-empty set, then the ***Fuzzy Multi Number*** (*FMN*) A on X is defined as
A= $\left\{\left(x, \mu_A^1(x), \mu_A^2(x), \dots \dots \dots \mu_A^n(x)\right) / x \in X\right\}$, where and this membership function maps each element of X to a membership value between 0 and 1∋ $\mu_A^1(x) \geq \mu_A^2(x) \geq, \dots \geq \mu_A^n(x)$ for $x \in X$.

2.7 Definition

S (***A***, ***B***) is said to be the ***Similarity Measure*** between A and B, where A, B ∈ X and X is an *FS*, as *S* (***A***, ***B***) satisfies the following properties

1. *S* (***A***, ***B***) ∈ [0,1]
2. *S* (***A***, ***B***) = 1 if and only if A = B
3. *S* (***A***, ***B***) = *S* (***B***, ***A***)

2.8 Definition

Let A = $\{\langle x_i, \mu_A(x_i)\rangle / x_i \in X\}$and B = $\{\langle x_i, \mu_B(x_i)\rangle / x_i \in X\}$ be two *FS*s on the finite universe of discourse X =$\{x_1, x_2, \dots \dots ., x_n\}$, then the correlation coefficient of A and B is the ***Fuzzy Correlation Measure*** defined as

$$\rho_{FS}(A,B) = \frac{C_{FS}(A,B)}{\sqrt{C_{FS}(A,A) \times C_{FS}(B,B)}}$$

where $C_{FS}(A,B) = \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)$ and
$C_{FS}(A,A) = \sum_{i=1}^{n} \mu_A(x_i)\mu_A(x_i)$,
$C_{FS}(B,B) = \sum_{i=1}^{n} \mu_B(x_i)\mu_B(x_i)$.

3. CORRELATION MEASURE FOR FUZZY MULTI NUMBERS

In this section, we have formulated the Correlation Measure for different types of *FMN*s such as triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, Nanogonal and decagonal. As, the correlation measure always lies between 0 and 1; the proposed measure was verified by some illustrative numerical examples based on the correlation measure properties.

3.1 Definition

Let X =$\{x_1, x_2, \dots \dots ., x_n\}$ be the finite universe of discourse and *A* and *B* are two *FMS*s on X, then the correlation coefficient of A and B is the ***Fuzzy Multi Correlation Measure*** defined on fuzzy multi numbers as

$$\rho_{FMN}(A,B) = \frac{C_{FMN}(A,B)}{\sqrt{C_{FMN}(A,A) \times C_{FMN}(B,B)}} \qquad (3.1)$$

3.2 Triangular Fuzzy Multi Numbers (a, b, c)

Let X =$\{x_1, x_2, \dots \dots ., x_n\}$ be the finite universe of discourse and the two *FMN*s in the universe be
A = $\left\{\left(a_1^i, b_1^i, c_1^i\right)\right\}$ and B = $\left\{\left(a_2^i, b_2^i, c_2^i\right)\right\}$, then equation 3.1 represents the correlation measure for triangular fuzzy multi numbers, where

$$\boldsymbol{C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\left\{(a_1^i \times a_2^i) + (b^i \times b^i) + (c_1^i \times c_2^i)\right\}\right]},$$

$$\boldsymbol{C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\left\{(a_1^i \times a_1^i) + (b^i \times b^i) + (c_1^i \times c_1^i)\right\}\right]} \textbf{ and}$$

$$\boldsymbol{C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\left\{(a_2^i \times a_2^i) + (b^i \times b^i) + (c_2^i \times c_2^i)\right\}\right]}.$$

3.2 Trapezoidal Fuzzy Multi Numbers (a, b, c, d)

Let X =$\{x_1, x_2, \dots \dots ., x_n\}$ be the finite universe of discourse and A = $\left\{\left(a_1^i, b_1^i, c_1^i, d_1^i\right)\right\}$ and
B = $\left\{\left(a_2^i, b_2^i, c_2^i, d_2^i\right)\right\}$, then using the equation 3.1 is the correlation measure for the trapezoidal fuzzy multi numbers where

$$\boldsymbol{C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\left\{(a_1^i \times a_2^i) + (b^i \times b^i) + (c^i \times c^i) + (d_1^i \times d_2^i)\right\}\right]}$$

$$\boldsymbol{C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\left\{(a_1^i \times a_1^i) + (b^i \times b^i) + (c^i \times c^i) + (d_1^i \times d_1^i)\right\}\right]} \textbf{ and}$$

$$C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b^i \times b^i) + (c^i \times c^i) + (d_2^i \times d_2^i)\}\right].$$

3.3 Pentagonal Fuzzy Multi Numbers (a,b,c,d,e)

Let X $=\{x_1, x_2, \dots\dots, x_n\}$ be the finite universe of discourse and A $= \{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i)\}$ and B $= \{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i)\}$, then equation 3.1 defines the correlation measure used for the pentagonal fuzzy multi numbers, where

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b_1^i \times b_2^i) + (c^i \times c^i) + (d_1^i \times d_2^i) + (e_1^i \times e_2^i)\}\right]$$

$$C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c^i \times c^i) + (d_1^i \times d_1^i) + (e_1^i \times e_1^i)\}\right] \textbf{ and}$$

$$C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c^i \times c^i) + (d_2^i \times d_2^i) + (e_2^i \times e_2^i)\}\right].$$

3.4 Hexagonal Fuzzy Multi Numbers(a,b,c,d,e,f)

Let X $=\{x_1, x_2, \dots\dots, x_n\}$ be the finite universe of discourse and A $= \{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i)\}$ and B $= \{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i, f_2^i)\}$, then by using the equation 3.1 is the correlation measure for the hexagonal fuzzy multi numbers, where

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b_1^i \times b_2^i) + (c^i \times c^i) + (d^i \times d^i) + (e_1^i \times e_2^i) + (f_1^i \times f_2^i)\}\right]$$

$$C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c^i \times c^i) + (d^i \times d^i) + (e_1^i \times e_1^i) + (f_1^i \times f_1^i)\}\right] \textbf{ and}$$

$$C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c^i \times c^i) + (d^i \times d^i) + (e_2^i \times e_2^i) + (f_2^i \times f_2^i)\}\right].$$

3.5 Heptagonal Fuzzy Multi Numbers(a,b,c,d,e,f,g)

Let X $=\{x_1, x_2, \dots\dots, x_n\}$ be the finite universe of discourse and A $= \{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i)\}$ and B $= \{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i, f_2^i, g_2^i)\}$, then the correlation measure for heptagonal fuzzy multi numbers is formulated from the equation 3.1, where

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d^i \times d^i) + (e_1^i \times e_2^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i)\}\right]$$

$$C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d^i \times d^i) + (e_1^i \times e_1^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i)\}\right] \textbf{ and}$$

$$C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + (d^i \times d^i) + (e_2^i \times e_2^i) + (f_2^i \times f_2^i) + (g_2^i \times g_2^i)\}\right].$$

3.6 Octagonal Fuzzy Multi Numbers(a,b,c,d,e,f,g,h)

Let X $=\{x_1, x_2, \dots\dots, x_n\}$ be the finite universe of discourse and the two *FMN*s in the universe be A $= \{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i, h_1^i)\}$ and B $= \{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i, f_2^i, g_2^i, h_2^i)\}$, then the equation 3.1 provides the mathematical expression for the correlation measure for the octagonal fuzzy multi numbers, where

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d^i \times d^i) + (e^i \times e^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i)\}\right]$$

$$C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d^i \times d^i) + (e^i \times e^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i)\}\right] \textbf{ and}$$

$$C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + (d^i \times d^i) + (e^i \times e^i) + (f_2^i \times f_2^i) + (g_2^i \times g_2^i) + (h_2^i \times h_2^i)\}\right].$$

3.7 Nanogonal Fuzzy Multi Numbers(a,b,c,d,e,f,g,h,i)

Let X $=\{x_1, x_2, \dots\dots, x_n\}$ be the finite universe of discourse and the two *FMN*s in the universe be A $= \{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i, h_1^i, i_1^i)\}$ and B $= \{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i, f_2^i, g_2^i, h_2^i, i_2^i)\}$, then the equation 3.1 is the correlation measure for the Nanogonal fuzzy multi numbers where

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e^i \times e^i) + (f_1^i \times f_2^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i)\}\right]$$

$$C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e^i \times e^i) + (f_1^i \times f_1^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i)\}\right]$$

$$C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + (d_2^i \times d_2^i) + (e^i \times e^i) + (f_2^i \times f_2^i) + (g_2^i \times g_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i)\}\right].$$

3.8 Decagonal Fuzzy Multi Numbers(a,b,c,d,e,f,g,h,i,j)

Let X $=\{x_1, x_2, \dots\dots, x_n\}$ be the finite universe of discourse and the two *FMN*s in the universe be A $= \{(a_1^i, b_1^i, c_1^i, d_1^i, e_1^i, f_1^i, g_1^i, h_1^i, i_1^i, j_1^i)\}$ and B $= \{(a_2^i, b_2^i, c_2^i, d_2^i, e_2^i, f_2^i, g_2^i, h_2^i, i_2^i, j_2^i)\}$, then the correlation measure applicable to decagonal fuzzy multi number of the equation 3.1 where,

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b_1^i \times b_2^i) + (c_1^i \times c_2^i) + (d_1^i \times d_2^i) + (e^i \times e^i) + (f^i \times f^i) + (g_1^i \times g_2^i) + (h_1^i \times h_2^i) + (i_1^i \times i_2^i) + (j_1^i \times j_2^i)\}\right]$$

$$C_{FMN}(A,A) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b_1^i \times b_1^i) + (c_1^i \times c_1^i) + (d_1^i \times d_1^i) + (e^i \times e^i) + (f^i \times f^i) + (g_1^i \times g_1^i) + (h_1^i \times h_1^i) + (i_1^i \times i_1^i) + (j_1^i \times j_1^i)\}\right]$$

$$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_2^i) + (b_2^i \times b_2^i) + (c_2^i \times c_2^i) + (d_2^i \times d_2^i) + (e^i \times e^i) + (f^i \times f^i) + (g_2^i \times g_2^i) + (h_2^i \times h_2^i) + (i_2^i \times i_2^i) + (j_2^i \times j_2^i)\}\right].$$

3.9 Proposition

The defined correlation measure $\rho_{FMN}(A,B)$, between the *FMN*s A and B satisfies the following properties

i. $0 \le \rho_{FMN}(A,B) \le 1$
ii. $\rho_{FMN}(A,B) = 1$ if and only if A = B
iii. $\rho_{FMN}(A,B) = \rho_{FMN}(B,A)$

Proof

i. $\mathbf{0 \le \rho_{FMN}(A,B) \le 1}$

As the defuzzification of the fuzzy numbers results the value in between real values, the correlation measure $\rho_{FMN}(A,B)$ lies between 0 and 1.

3.9.1 Example

Let A= {(2,4,5), (3,5,6), (2,4,7)} and
B ={(3,5,7), (3,4,8), (4,5,9)}
Here, the cardinality $\eta = 3$ as
$C_{FMN}(A,B) = 76.33$, $C_{FMN}(A,A) = 67.33$ and $C_{FMN}(B,B)$ =92

$$\rho_{FMN}(A,B) = \frac{76.33}{\sqrt{(67.33)\times(92)}} = 0.9698$$

ii. $\boldsymbol{\rho_{FMN}(A,B) = 1}$ **if and only if A = B**

a. Let the two *FMN*s A and B be equal (i.e.) A=B.

Hence $C_{FMN}(A,A) = C_{FMN}(B,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b^i \times b^i) + (c_1^i \times c_1^i)\}\right]$

$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b^i \times b^i) + (c_1^i \times c_2^i)\}\right]$

$= \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_1^i) + (b^i \times b^i) + (c_1^i \times c_1^i)\}\right] = C_{FMN}(A,A)$

Hence $\rho_{FMN}(A,B) = \frac{C_{FMN}(A,B)}{\sqrt{C_{FMN}(A,A)\times C_{FMN}(B,B)}} = \frac{C_{FMN}(A,A)}{\sqrt{C_{FMN}(A,A)\times C_{FMN}(A,A)}} = 1$

b. Let the $\rho_{FMN}(A,B) = 1$

The unit measure is possible only if $\rho_{FMN}(A,B) = \frac{C_{FMN}(A,B)}{\sqrt{C_{FMN}(A,A)\times C_{FMN}(B,B)}} = 1$

This refers that $C_{FMN}(A,B) = C_{FMN}(A,A) = C_{FMN}(B,B)$.
Hence A=B.

3.9.2 Example

Let A={(5,6,10), (8,9,11), (7,8,12), (7,9,14)} and
B={(5,6,10), (8,9,11), (7,8,12), (7,9,14)}
Here, the cardinality $\eta = 4$ as $C_{FMN}(A,B) = 252.5$, $C_{FMN}(A,A) = 252.5$ and $C_{FMN}(B,B) = 252.5$

$$\rho_{FMN}(A,B) = \frac{252.5}{\sqrt{(252.5)\times(252.5)}} = 1$$

iii. $\boldsymbol{\rho_{FMN}(A,B) = \rho_{FMN}(B,A)}$

It is obvious that $\rho_{FMN}(A,B) = \frac{C_{FMN}(A,B)}{\sqrt{C_{FMN}(A,A)\times C_{FMN}(B,B)}} = \frac{C_{FMN}(B,A)}{\sqrt{C_{FMN}(A,A)\times C_{FMN}(A,A)}} = \rho_{FMN}(B,A)$ as

$C_{FMN}(A,B) = \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_1^i \times a_2^i) + (b^i \times b^i) + (c_1^i \times c_2^i)\}\right]$

$= \frac{1}{\eta}\left[\sum_{i=1}^{\eta}\{(a_2^i \times a_1^i) + (b^i \times b^i) + (c_2^i \times c_1^i)\}\right] = C_{FMN}(B,A)$.

3.9.3 Example

Let A={(4,6,7), (5,6,9), (4,5,12)} and
B={(2,5,9), (2,5,12), (3,6,11)}
Here, the cardinality $\eta = 3$
as $C_{FMN}(A,B) = 135$, $C_{FMN}(B,A) = 135.33$ $C_{FMN}(A,A) = 142.33$ and $C_{FMN}(B,B) = 131.33$

$$\rho_{FMN}(A,B) = \frac{135}{\sqrt{(142.33)\times(135.33)}} = 0.9727$$

Similarly, $\rho_{FMN}(B,A) = \frac{135}{\sqrt{(135.33)\times(142.33)}} = 0.9727$

3.10 Remark

We have verified the proposition for Triangular Fuzzy Multi Numbers along with the numerical examples. The same conditions can be applied for trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, Nanogonal and decagonal *FMNs*, and we are sure that they all will pertain the propositions of proposed measure mentioned in this section.

4. NUMERICAL EVALUATION

In this section, few numerical examples were demonstrated to validate the proposed Correlation Measure for the *FMN*s. The examples clearly show that the correlation value for *FMN*s lie between 0 and 1 for all the categories of triangle, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, Nanogonal and decagonal *FMN*s.

4.1 Triangular Fuzzy Multi Numbers

A triangular fuzzy multi number is denoted by 3 – tuples (a^i, b^i, c^i), where a^i, b^i and c^i are real numbers and
$a^i \leq b^i \leq c^i$ with membership function defined.

4.1.1 Example

Let A = {(12,20,50), (16,20,59), (12,25,65), (19,20,70), (19,25,65)} and
B={(20,25,28), (23,25,29), (23,26,31), (24,25,27), (20,26,30)}.

Here, the cardinality $\eta = 5$ and found to be equal as $C_{FMN}(A,B) = 2697.2$, $C_{FMN}(A,A) = 4609.4$ and $C_{FMN}(B,B) = 1975.2$.

$$\rho_{FMN}(A,B) = \frac{2697.2}{\sqrt{(4609.4) \times (1975.2)}} = 0.8939$$

4.1.2 Example

Let A = {(2,5,6), (3,6,7), (2,5,7), (3,4,6)} and
B={(4,6,9)}

Here, the cardinality $\eta = max\{\eta(A), \eta(B)\} = max\{4,1\} = 4$ and it was found to be unequal as $C_{FMN}(A,B) = 98.5$, $C_{FMN}(A,A) = 74.5$ and $C_{FMN}(B,B) = 133$.

$$\rho_{FMN}(A,B) = \frac{98.5}{\sqrt{(74.5) \times (133)}} = 0.9895$$

4.2 Trapezoidal Fuzzy Multi Numbers

A **trapezoidal fuzzy multi number** is denoted by 4 – tuples (a^i, b^i, c^i, d^i) where a^i, b^i, c^i and d^i are real numbers and $a^i \leq b^i \leq c^i \leq d^i$ with membership function defined.

4.2.1 Example

Let A = {(1,3,4,5), (2,3,5,6), (1,3,5,7)} and
B={(2,5,7,8), (3,4,5,9), (3,5,6,12)}.

Here, the cardinality $\eta = 3$ and found to be equal as per $C_{FMN}(A,B) = 104.6$, $C_{FMN}(A,A) = 69.66$ and $C_{FMN}(B,B) = 163.33$.

$$\rho_{FMN}(A,B) = \frac{104.6}{\sqrt{(69.66) \times (163.33)}} = 0.9840.$$

4.2.2 Example

Let A = {(12,13,15,17), (13,14,15,18), (12,13,15,19), (13,14,15,20)} and
B={(20,28,29,30)}

Here, the cardinality $\eta = max\{\eta(A), \eta(B)\} = max\{4,1\} = 4$ and the cardinality value is unequal as per $C_{FMN}(A,B) = 1618$, $C_{FMN}(A,A) = 907.5$ and $C_{FMN}(B,B) = 2925$

$$\rho_{FMN}(A,B) = \frac{1618}{\sqrt{(907.5) \times (2925)}} = 0.9931.$$

4.3 Pentagonal Fuzzy Multi Numbers

A **pentagonal fuzzy multi number** A is denoted by 5 – tuples $(a^i, b^i, c^i, d^i, e^i)$ where a^i, b^i, c^i, d^i and e^i are real numbers and $a^i \leq b^i \leq c^i \leq d^i \leq e^i$ with membership function is clear.

4.3.1 Example

Let A= {(11,12,14,17,19), (12,13,14,19,20), (11,12,13,16,20), (13,14,15,17,21), (12,13,14,20,22)} and
B ={(22,23,27,28,29), (23,24,28,29,30), (22,25,27,28,32), (23,26,28,29,34), (24,25,27,29,37)}

Here, the cardinality $\eta = 5$ and found to be equal as per $C_{FMN}(A,B) = 2138.2$, $C_{FMN}(A,A) = 1236.8$ and $C_{FMN}(B,B) = 3753.8$

$$\rho_{FMN}(A,B) = \frac{2138.2}{\sqrt{(1236.8) \times (3753.8)}} = 0.9923.$$

4.3.2 Example

Let A = {(12,13,14,16,17), (11,12,13,17,19), (12,13,14,19,22)} and
B={(22,26,29,31,32)}

Here, the cardinality $\eta = max\{\eta(A), \eta(B)\} = max\{3,1\} = 3$ and found to be unequal as $C_{FMN}(A,B) = 2138.3$, $C_{FMN}(A,A) = 1164$ and $C_{FMN}(B,B) = 3986$

$$\rho_{FMN}(A,B) = \frac{2138.3}{\sqrt{(1164) \times (3986)}} = 0.9927.$$

4.4 Hexagonal Fuzzy Multi Numbers

A **hexagonal fuzzy multi number** A is specified by 6 tuples $(a^i, b^i, c^i, d^i, e^i, f^i)$ where a^i, b^i, c^i, d^i, e^i and f^i are real number and $a^i \leq b^i \leq c^i \leq d^i \leq e^i \leq f^i$ with membership function.

4.2.1 Example

Let A = {(2,4,5,6,9,10), (3,5,6,7,8,11), (3,4,6,7,10,12), (2,5,6,8,12,14)} and
B={(14,16,18,20,21,22), (13,14,19,20,22,23), (15,16,20,22,23,24), (17,18,20,21,26,29)}.

Here, the cardinality $\eta = 4$ and found to be equal by way of $C_{FMN}(A,B) = 893.5, C_{FMN}(A,A) = 347.25$ and $C_{FMN}(B,B) = 2420.25$

$$\rho_{FMN}(A,B) = \frac{893.5}{\sqrt{(347.25)\times(2420.25)}} = 0.9746.$$

4.2.2 Example

Let A = {(1,3,4,6,8,10), (2,5,7,9,11,13), (1,4,8,10,12,15)} and
B = {(8,9,10,11,12,13)}

Here, the cardinality $\eta = max\{\eta(A), \eta(B)\} = max\{3,1\} = 3$ and found to be unequal
as $C_{FMN}(A,B) = 490.3$, $C_{FMN}(A,A) = 408.3$ and $C_{FMN}(B,B) = 679$

$$\rho_{FMN}(A,B) = \frac{490.3}{\sqrt{(408.3)\times(679)}} = 0.9315.$$

4.5 Heptagonal Fuzzy Multi Numbers

The **heptagonal fuzzy multi number** denoted by $(a^i, b^i, c^i, d^i, e^i, f^i, g^i)$ where, $a^i, b^i, c^i, d^i, e^i, f^i$ and g^i are real number and $a^i \le b^i \le c^i \le d^i \le e^i \le f^i \le g^i$ with its membership function.

4.5.1 Example

Let A = {(4,6,8,11,14,16,18), (5,7,9,12,13,17,19), (8,9,10,12,15,19,22), (4,8,9,13,19,23,25)} and
B=(25,26,27,28,30,31,32), (26,27,28,29,32,34,36), (25,26,27,29,33,35,37), (26,27,28,30,35,39,43)}.Here, the cardinality $\eta = 4$ and found to be equal
as per $C_{FN}(A,B) = 2873.5, C_{FN}(A,A) = 1358.75$ and $C_{FN}(B,B) = 6610.75$

$$\rho_{FMN}(A,B) = \frac{2873.5}{\sqrt{(1358.75)\times(6610.75)}} = 0.9588$$

4.5.2 Example

Let A={(3,4,5,6,7,8,9), (2,3,4,7,8,10,12), (3,4,5,6,12,13,15), (2,4,5,9,13,17,19), (3,4,5,10,15,17,21)}
and B={(21,23,24,25,27,28,29)}

Here, the cardinality $\eta = max\{\eta(A), \eta(B)\} = max\{5,1\} = 5$ and found to be unequal
as $C_{FMN}(A,B) = 1549.8$, $C_{FMN}(A,A) = 663$ and $C_{FMN}(B,B) = 4525$.

$$\rho_{FMN}(A,B) = \frac{1549.8}{\sqrt{(663)\times(4525)}} = 0.8948.$$

4.6 Octagonal Fuzzy Multi Numbers

The **octagonal fuzzy multi number** denoted by $(a^i, b^i, c^i, d^i, e^i, f^i, g^i, h^i)$ where, $a^i, b^i, c^i, d^i, e^i, f^i, g^i$and $h^i$are real number and $a^i \le b^i \le c^i \le d^i \le e^i \le f^i \le g^i \le h^i$ with its membership function.

4.6.1 Example

Let A = {(1,3,5,7,9,12,15,17), (2,4,7,9,10,15,16,19), (1,4,6,9,11,16,17,20)} and
B={(20,21,22,23,27,28,29,30), (22,23,24,27,28,29,31,32), (20,23,25,26,27,30,32,34)}.

Here the cardinality $\eta = 3$ and found to be equal
as $C_{FMN}(A,B) = 2247, C_{FMN}(A,A) = 1038.3$ and $C_{FMN}(B,B) = 5691.6$

$$\rho_{FMN}(A,B) = \frac{2247}{\sqrt{(1038.3)\times(5691.6)}} = 0.9243$$

4.6.2 Example

Let A={(2,7,9,11,12,13,15,17), (3,4,7,8,10,11,13,15), (2,5,7,9,11,13,15,17), (4,6,8,10,12,14,18,20)}
and B={(20,23,27,28,29,30,31,32)}

Here, the cardinality $\eta = max\{\eta(A), \eta(B)\} = max\{4,1\} = 4$ and found to be unequal
by means of $C_{FMN}(A,B) = 2387.25$, $C_{FMN}(A,A) = 1019.5$ and $C_{FMN}(B,B) = 6168$

$$\rho_{FMN}(A,B) = \frac{2387.25}{\sqrt{(1019.5)\times(6168)}} = 0.9519$$

4.7 Nanogonal Fuzzy Multi Numbers

The **Nanogonal fuzzy multi number** denoted by $(a^i, b^i, c^i, d^i, e^i, f^i, g^i, h^i, i^i)$ where $a^i, b^i, c^i, d^i, e^i, f^i, g^i, h^i$ and i^i real number and $a^i \le b^i \le c^i \le d^i \le e^i \le f^i \le g^i \le h^i \le i^i$ with its membership function.

4.7.1 Example

Let A = {(3,6,9,10,12,13,14,17,19), (2,4,6,8,10,11,13,15,17), (4,8,9,10,12,15,17,19,21)} and
B={(1,3,4,7,12,15,16,18,20), (2,3,5,8,10,11,13,15,19), (1,5,9,11,12,14,16,18,20)}.

Here, the cardinality $\eta = 3$ and found to be equal
as $C_{FMN}(A,B) = 1349, C_{FMN}(A,A) = 1391.3$ and $C_{FMN}(B,B) = 1415.3$

$$\rho_{FMN}(A,B) = \frac{1349}{\sqrt{(1391.3)\times(1415.3)}} = 0.9613$$

4.7.2 Example
Let A = {(7,8,9,10,11,12,13,14,15), (8,9,11,13,15,17,19,20,21), (7,9,10,12,13,15,16,19,20)}
and B={(9,12,14,15,17,20,22,24,28)} .
Here, the cardinality $\eta = max\{\eta(A),\eta(B)\} = max\{3,1\} = 3$ and found to be unequal
as per $C_{FMN}(A,B) = 2298, C_{FMN}(A,A) = 1695$ and $C_{FMN}(B,B) = 3179$

$$\rho_{FMN}(A,B) = \frac{2298}{\sqrt{(1695)\times(3179)}} = 0.9899$$

4.8 Decagonal Fuzzy Multi Numbers
The **Decagonal fuzzy multi number** denoted by $(a^i, b^i, c^i, d^i, e^i, f^i, g^i, h^i, i^i, j^i)$ where $a^i, b^i, c^i, d^i, e^i, f^i, g^i, h^i, i^i$ and j^i are real number and $a^i \le b^i \le c^i \le d^i \le e^i \le f^i \le g^i \le h^i \le i^i \le j^i$ with its membership function.

4.8.1 Example
Let A = {(1,2,3,5,6,8,10,11,13,15), (2,3,8,9,10,11,13,15,17,19)}
and B={(8,9,10,12,14,16,18,20,22,24), (10,12,15,17,18,19,20,21,23,25)}.
Here, the cardinality $\eta = 2$ and found to be equal
as $C_{FMN}(A,B) = 1766.5, C_{FMN}(A,A) = 1088.5$ and $C_{FMN}(B,B) = 3031.5$

$$\rho_{FMN}(A,B) = \frac{1766.5}{\sqrt{(1088.5)\times(30311.5)}} = 0.9725$$

4.8.2 Example
Let A = {(5,6,8,10,12,13,14,17,19,20), (4,5,9,10,11,12,15,18,20,22)}
and B={(9,11,13,15,17,18,20,22,23,25)}
Here, the cardinality $\eta = max\{\eta(A),\eta(B)\} = max\{2,1\} = 2$ and found to be unequal
as $C_{FMN}(A,B) = 2430.5, C_{FMN}(A,A) = 1852$ and $C_{FMN}(B,B) = 3247$

$$\rho_{FMN}(A,B) = \frac{2430.5}{\sqrt{(11852)\times(3247)}} = 0.972$$

Table 4.1 Consolidation of Numerical Examples

Types of Fuzzy Multi Numbers	Equal Cardinality	Unequal Cardinality
Triangular	0.8939	0.9895
Trapezoidal	0.9840	0.9931
Pentagonal	0.9923	0.9927
Hexagonal	0.9746	0.9315
Heptagonal	0.9588	0.8948.
Octagonal	0.9243	0.9519
Nanogonal	0.9613	0.9899
Decagonal	0.9725	0.9725

From all the above numerical examples, it is clear that the correlation measure for all the categories of *FMN*s for equal and unequal cardinality lies between 0 and 1.

CONCLUSION
The Correlation Measure of *FMN*s from *IFMS*s theory was proposed and derived in this paper. The prominent characteristic of this method was the correlation measure of any two *FMN*s equals to one if and only if the two *FMN*s are the same. From the numerical evaluation, it is clear that this proposed measure can be applied to any decision-making problems and in future this concept will be applied in medical diagnosis, signature forgery detection and pattern recognition.

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