INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD)
© 2026 IJNRD | Volume 11, Issue 1, January 2026 | ISSN: 2456-4184 | IJNRD.ORG

A Hybrid Digital Twin for Reheating Furnace Optimization through a Data-Physics Fusion for Emerging Industries

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Abstract: Background: Industrial reheating furnaces are among the most energy-intensive units in secondary steel production, contributing significantly to operational costs and greenhouse gas emissions, especially in emerging economies where legacy systems dominate.
Objective: This study aims to design, validate, and demonstrate a novel hybrid Digital Twin framework to optimize energy efficiency and reduce emissions in reheating furnaces, tailored for resource-constrained industrial settings.
Methodology: A five-layer hybrid Digital Twin architecture is proposed, integrating physics-based thermodynamic modeling with machine learning (XGBoost) through a dynamic confidence-weighted fusion mechanism. The framework is developed using a two-year energy audit dataset from a steel plant and validated through a rigorous two-step process: calibration on 2023 data and independent prediction of 2024 performance.
Results: The hybrid model achieved exceptional predictive accuracy, with only −0.5% deviation in annual fuel consumption on unseen data. Simulation results indicate that optimizing production scheduling to maximize hot-charge utilization can reduce Specific Energy Consumption by ~36%, saving over 179,000 liters of heavy fuel oil and preventing more than 560 tons of CO_2e emissions annually per furnace.
Conclusion: This research presents a novel, scalable, and economically viable Digital Twin framework that enables emerging industries to leapfrog toward sustainable manufacturing. The study demonstrates that significant energy and emission reductions are achievable through intelligent process integration, offering a practical roadmap for the adoption of Industry 4.0 technologies in developing industrial contexts.

Keywords **- Digital Twin, Reheating Furnace, Energy Optimization, Smart Manufacturing, Sustainable Production, Cyber-Physical Systems.**

1. INTRODUCTION

1. Introduction

1.1. Global Context and Industrial Challenge

The global steel industry, responsible for approximately 7% of global anthropogenic CO_2 emissions, stands at a critical juncture between sustaining economic development and achieving climate targets [1]. Secondary steel production, particularly in growing economies, often relies on energy-intensive reheating furnaces (RHFs). These units thermally condition billets for rolling, typically consuming heavy fuel oil (HFO) or natural gas. Studies indicate that RHFs can account for 30–50% of a mini-mill's total energy use, with thermal efficiencies frequently below 50% due to radiant, convective, and exhaust losses [2]. The operational inefficiency is compounded by sub-optimal production scheduling, where the underutilization of direct hot-charge (HC) pathways leads to the complete waste of embodied heat in cast billets, necessitating full reheating from ambient temperature [3].

1.2. The Leapfrogging Imperative and Technological Gap

Emerging economies, such as Ethiopia, face the dual imperative of expanding industrial capacity while committing to sustainable development under frameworks like the Nationally Determined Contributions (NDCs) to the Paris Agreement. The Government of Ethiopia's *Ten-Year Development Plan* explicitly targets the transformation of the manufacturing sector through technology adoption and enhanced productivity, aligning with aspirations for an Industry 4.0 ecosystem [4]. However, a significant implementation gap exists. Legacy manufacturing assets, like traditional RHFs, often operate with limited sensorization, fixed control setpoints, and siloed production planning. This results in a reactive, experience-based operational paradigm non-suitable for maximizing efficiency or integrating with advanced smart grids and renewable energy systems [5].

1.3. Digital Twins as a Strategic Enabler

Digital Twin (DT) technology—a virtual, dynamic representation of a physical system that updates and evolves using real-time data—has emerged as a cornerstone of Industry 4.0. It enables simulation, predictive analysis, and autonomous control, bridging the gap between the physical and digital worlds [6]. While DTs have seen advanced application in sectors like aerospace and automotive for predictive maintenance and design optimization, their deployment for *operational energy and emission optimization of legacy thermal processes in industrializing contexts* remains limited. Current research often focuses on either high-fidelity physics-based model requiring extensive computational resources or purely data-driven models dependent on large, clean historical datasets—both assumptions rarely valid in the target context [7, 8].

1.4. Research Objective and Novel Contribution

This paper addresses the identified gap by proposing, detailing, and validating a novel hybrid Digital Twin framework specifically engineered for the RHF optimization challenge in leapfrogging industrial settings. The primary objectives are:

1. To design a scalable, five-layer DT architecture that synergistically combines explainable physics-based models with adaptive data-driven algorithms, beginning with available audit data.
2. To demonstrate the framework's application and quantify its impact through a validated case study, showing significant efficiency and emission gains achievable through operational scheduling changes.
3. To provide a clear, phased implementation roadmap that aligns with the resource constraints and strategic goals of industries in developing economies.

The novelty of this work lies in its context-aware methodology: it provides a structured pathway from basic energy audit data to a sophisticated operational decision-support tool, explicitly designed to unlock value in legacy systems without presupposing advanced infrastructure.

2. Literature Review

2.1. Energy Dynamics and Inefficiencies in Reheating Furnaces

Reheating furnaces operate on the principle of raising steel billets to a uniform temperature (typically 1150–1250°C) suitable for plastic deformation in rolling mills. The energy input is predominantly consumed by: (i) heating the steel stock, (ii) compensating for heat losses through furnace walls and hearth, (iii) heating combustion air, and (iv) the enthalpy of exhaust gases. The Specific Energy Consumption (SEC), measured in Gigajoules per tonne (GJ/t), is the key performance indicator. International benchmarks suggest a best-practice SEC range of 1.6–2.2 GJ/t for modern, well-insulated furnaces with optimized combustion [9]. In contrast, inefficient operations, often characterized by improper air-to-fuel ratios, poor refractory condition, and—critically—non-optimized charging schedules, can exhibit SEC values exceeding 4.0 GJ/t [10]. The work of Hasanbeigi et al. [11] systematically categorizes these energy efficiency opportunities, highlighting process integration (like HC utilization) as a major, often low-cost measure.

2.2. Evolution and Typology of Digital Twins

The DT concept, first formally articulated by Grieves [12], has evolved from a product lifecycle management tool to a core component of cyber-physical production systems. A widely accepted taxonomy by Kritzinger et al. [13] distinguishes between:

- Digital Model: A digital representation with no automatic data exchange.
- Digital Shadow: A unidirectional automated data flow from physical to digital.
- Digital Twin: A fully integrated, bidirectional flow where the physical entity drives the digital state and *vice versa*.

For complex, non-linear thermal processes like reheating, the most robust approach is the Hybrid Twin [14]. This paradigm merges physics-based "white-box" models (grounded in first principles like mass and energy balances) with data-driven "black-box" models (e.g., machine learning algorithms). The physics model ensures robustness and extrapolation capability, while the data-driven component continuously calibrates the system to actual operating conditions, capturing unmodeled dynamics and degradation.

2.3. Digital Twins in Industrial Energy and Sustainability

The application of DTs in manufacturing is broadening. Schluse et al. [15] demonstrated their use for real-time factory simulation and virtual commissioning. In the energy domain, DTs are applied to optimize building HVAC systems and district heating networks [16]. For industrial furnaces, research is emerging. Liu et al. [17] developed a DT for a slab reheating furnace using deep reinforcement learning to optimize heating curves, reducing fuel consumption by ~5%. Similarly, Zhou et al. [18] created a physics-informed neural network model for a walking beam furnace to predict billet temperature. A common thread in these studies is their focus on **control optimization** within a given production schedule, often assuming a high degree of existing automation and data availability.

2.4. Identified Research Gap

A salient gap persists in the literature concerning practical, scalable DT frameworks for operational scheduling and process integration optimization in legacy industrial settings within developing economies. These environments are characterized by constrained capital for sensor retrofits, intermittent data historization, and an urgent need for solutions with rapid, demonstrable return on investment (ROI). While advanced studies focus on perfecting control for a fixed schedule, the lower-hanging fruit—optimizing the schedule itself to minimize furnace use—is less explored through a DT lens, particularly in a resource-constrained context. This paper aims to fill this gap by presenting a DT methodology that starts with the analysis of production scheduling (a major inefficiency source) and scales towards advanced control, using a hybrid modeling approach tailored for data-scarce initial stages.

3. Methodology: The Hybrid Digital Twin Framework

The proposed framework is architected for practical, phased deployment. It is structured into five distinct but interconnected layers, each with a specific function, ensuring scalability from a basic simulation tool to a full cyber-physical system.

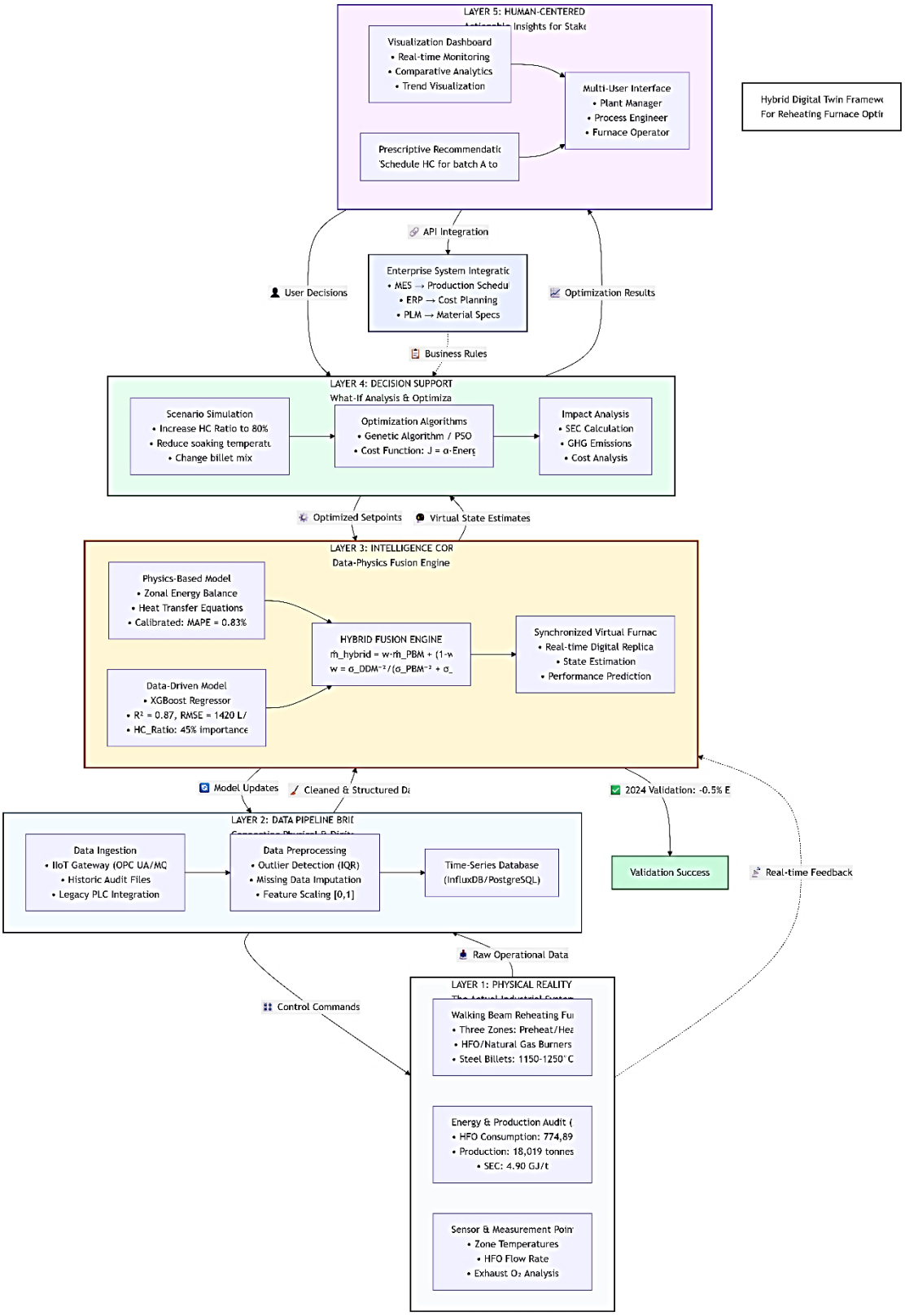

Figure 1: The Hybrid Digital Twin Framework

3.1. Layer 1: Physical Layer and Data Foundation

This layer encompasses the actual RHF system and the foundational data required to mirror it. For the initial DT instantiation, the primary data source is a comprehensive energy audit, which provides a static but rich dataset.

- **Key Audit Data Parameters:**
  - Production Data: Monthly tonnage of billets produced via continuous casting (Hot Charge source) and tonnage rolled via the RHF.

- Energy Data: Monthly HFO consumption (in liters), correlated with RHF production. Electrical energy consumption for auxiliary systems (pumps, fans).
- Operational Parameters: Furnace operating hours, average billet discharge temperature (if recorded), and basic production schedules.

- **Physical Parameters to be Mirrored:** For future real-time integration, the following physical parameters are identified as critical for sensing:
  - Thermal: Zone temperatures (preheat, heating, soaking), billet surface temperature (pyrometer), exhaust gas temperature and composition (O_2 analyzer).
  - Flow: HFO flow rate, combustion air flow rate.
  - Operational: Billet charging/discharging rate, furnace pressure.

3.2. Layer 2: Data Acquisition, Integration, and Preprocessing Layer

This layer manages the data pipeline from source to model.

- For Audit-Based Initiation: Historical audit data (e.g., from spreadsheets) is cleansed, formatted, and ingested into a time-series database (e.g., InfluxDB). This involves handling missing entries, correcting unit inconsistencies, and aligning production and energy data timestamps.

- For Real-Time Evolution: The system is designed to integrate with new sensor inputs via an Industrial IoT (IIoT) gateway (e.g., using OPC UA or MQTT protocols). A key function is data fusion, synchronizing data streams from legacy PLCs and new wireless sensors.
- Preprocessing Module: Executes critical steps before model ingestion:
  1. Outlier Detection: Using statistical methods (IQR) or model-based techniques to identify and filter erroneous sensor readings.
  2. Imputation: For missing data points in a time series, employs linear interpolation or forward-fill based on data stationarity.
  3. Normalization/Scaling: Scales numerical features (e.g., temperature, flow rates) to a standard range (e.g., [0,1]) to stabilize the training of data-driven model components.

3.3. Layer 3: The Hybrid Model Layer (Core Analytical Engine)

This layer hosts the integrated modeling intelligence, the novel core of the framework. The hybrid approach mitigates the "cold start" problem of purely data-driven models and the inflexibility of pure physics models in dynamic industrial environments.

3.3.1. Physics-Based Model (PBM): A Zonal Steady-State Thermodynamic Framework

The PBM decomposes the continuous reheating process into discrete, manageable control volumes (zones) to perform mass and energy balances, providing a stable, interpretable baseline.

- Zonal Discretization: The walking-beam RHF is modeled as three primary zones in series:

  1. Preheating Zone (Z1): Billets are heated from ambient charge temperature (T_{charge}) to approximately 800°C, primarily via convection from counter-flowing exhaust gases.

  2. Heating Zone (Z2): The main combustion zone where billets are heated to near-target temperature (~1150°C) by radiant heat from burners and hot refractory walls.

  3. Soaking Zone (Z3): Billets achieve thermal uniformity; temperature gradients are minimized before discharge at $T_{discharge}$ (target: 1250°C).

- Fundamental Governing Equations: For each zone i, a steady-state energy balance is applied:

$$\dot{Q}_{fuel,i} + \dot{Q}_{gas,in,i} = \dot{Q}_{stock,i} + \dot{Q}_{loss,i} + \dot{Q}_{gas,out,i} \qquad \textit{Equation 1}$$

Where:

- $\dot{Q}_{fuel,i}$: Heat input from HFO combustion. Calculated as:

$$\dot{Q}_{fuel,i} = \dot{m}_{HFO,i} \times NCV_{HFO} \qquad \textit{Equation 2,}$$

  where $NCV_{HFO} = 40.4$ MJ/kg [19] and $\dot{m}_{HFO,i}$ is the fuel mass flow rate allocated to the zone.
- $\dot{Q}_{gas,in,i}$: Sensible enthalpy of incoming gas stream (combustion products from upstream zone or fresh air for Z1).

- $\dot{Q}_{stock,i}$: Heat absorbed by the steel billets. This is the primary useful output:

$$\dot{Q}_{stock,i} = \dot{m}_{steel} \times \int_{T_{in,i}}^{T_{out,i}} C_p\,(T)\,dT \qquad \textit{Equation 3}$$

Here, $\dot{m}_{steel}$ is the billet mass flow rate (kg/s). The specific heat capacity of steel, $C_p(T)$, is modeled as a polynomial function of temperature.

- $\dot{Q}_{loss,i}$: Total heat loss through furnace walls, doors, and hearth. Estimated as:

$$\dot{Q}_{loss,i} = U_i \times A_i \times (T_{inner,i} - T_{ambient}) \qquad \textit{Equation 4}$$

U_i is the overall heat transfer coefficient of the refractory lining (W/m²·K), A_i is the surface area of zone i, and $T_{inner,i}$ is the average internal zone temperature.

- $\dot{Q}_{gas,out,i}$: Sensible enthalpy of the exiting gas stream, which becomes $\dot{Q}_{gas,in,i+1}$ for the next zone.

- **Calibration with Audit Data:** The model's unknown parameters, specifically the zonal heat loss coefficients (U_i) and the fuel distribution profile across zones, are calibrated using the annual 2023 audit data. An optimization routine (e.g., a gradient-based solver) adjusts these parameters to minimize the difference between the model's predicted total annual HFO consumption and the actual recorded value of 774,899 liters.

3.3.2. Data-Driven Model (DDM): An Adaptive Operational Performance Predictor

The DDM captures complex, non-linear relationships and transient effects not explicitly modeled in the steady-state PBM.

- **Feature Engineering & Dataset Construction:** From the historical audit and operational logs, a time-series dataset is constructed. Key engineered features include:
  - RHF-Through put: Billets rolled via RHF (tones/day).
  - HC Ratio: Proportion of daily production from Hot Charge (0 to 1).
  - Avg Billet Size: Average cross-sectional area (mm²).
  - Target Discharge Temp: Setpoint for billet discharge.
  - Ambient Temp: Local daily average temperature (°C).
  - Day Of Week: Encoded cyclically.
- **Model Selection & Architecture:** An Extreme Gradient Boosting (XGBoost) Regressor is employed due to its superior handling of tabular data, robustness to multicollinearity, and built-in feature importance metrics [20]. The target variable is Daily HFO Consumption (L).
  - Hyperparameters (Example): n-estimators=200, max-depth=6, learning rate=0.05, subsample=0.8.
  - Training Protocol: The 2023 data (70%) is used for training. The model learns to map the feature vector X to the fuel consumption target y.
- **Validation & Explainability:** The trained model's performance is validated on the 2023 test set (30%) and the entire 2024 dataset. A key output is Feature Importance, quantifying which operational factors most significantly drive fuel consumption.

3.3.3. Hybridization via a Dynamic Confidence-Weighted Ensemble

A dynamic fusion mechanism is proposed, based on the estimated uncertainty of each model's prediction.

1. **Uncertainty Estimation:**
   - For the **PBM**, the primary uncertainty σ_{PBM} is derived from the residuals of the calibration process.
   - For the **DDM**, the prediction uncertainty σ_{DDM} is estimated using the **jackknife+ method** or ensemble variance.
2. **Dynamic Weight Calculation:** The weight for the PBM, w, for a given prediction instance is calculated inversely proportional to its variance relative to the DDM's:

$$w = \frac{\sigma_{DDM}^{-2}}{\sigma_{PBM}^{-2} + \sigma_{DDM}^{-2}} \qquad \textit{Equation 5}$$

Consequently, the hybrid prediction is:

$$\dot{m}_{HFO,hybrid} = w \cdot \dot{m}_{HFO,PBM} + (1 - w) \cdot \dot{m}_{HFO,DDM} \qquad \textit{Equation 6}$$

3. **Feedback Loop for Model Evolution:** As new operational data streams in from Layer 2, the DDM is **retrained incrementally** (online learning) at scheduled intervals. This allows the hybrid model to adapt to long-term changes, ensuring the DT's accuracy decays over time.

3.4. Layer 4: Simulation, Optimization, and Service Layer

This layer is the "what-if" analysis engine and optimization kernel.

- **Scenario Simulator:** Users define scenarios (e.g., "Increase HC routing to 80%"). The simulator calls the hybrid model to execute these scenarios over a defined horizon, calculating resultant KPIs.

- **Optimization Module:** For automated strategy generation, an optimization algorithm (e.g., a genetic algorithm) is integrated. It searches the operational decision space to minimize a cost function J:

$$J = \alpha.(Energy\ Cost) + \beta.(GHG\ Emissions) + \gamma.(Throughput\ penality) \qquad \textit{Equation 7}$$

Where α, β, γ are user-defined weights reflecting business priorities.

- **GHG Calculator:** An integrated module uses the simulated HFO consumption and applies the latest IPCC emission factors to compute total CO_2 equivalent emissions [19].

3.5. Layer 5: Application and Visualization Layer

This layer delivers actionable insights to end-users.

- **Dashboard:** A web-based interface display:
  - Real-Time Monitors: Current furnace state, fuel rate, efficiency indicators.
  - Scenario Configuration Panel: Forms to set up and launch simulation studies.
  - Visual Analytics: Comparative bar charts of energy use, cost, and emissions across scenarios.
  - Prescriptive Recommendations: The system displays top-ranked optimization strategies (e.g., "Schedule billet batch A for HC tomorrow AM to save X liters of HFO").

4. Case Study Application, Validation, and Results

4.1. Case Study Baseline: Data Sourcing and Preprocessing

The case study is based on a 24-month detailed energy audit (January 2023 - December 2024) of a steel manufacturing plant. The primary data source was a 24-month detailed energy audit (January 2023 - December 2024), providing a robust dataset for model development and validation.

4.1.1. **Data Acquisition and Characteristics:** Daily production logs and monthly HFO consumption data were collected. Temporal misalignment was resolved by disaggregating monthly HFO consumption proportionally to daily RHF throughput.

4.1.2. **Baseline Performance Calculation and Benchmarking:** Specific Energy Consumption (SEC) was calculated for 2023 and 2024. The SEC for 2023 was 4.90 GJ/t and for 2024 was 3.96 GJ/t. Both values remained 52-88% higher than the global best-practice benchmark of 2.6 GJ/t.

Table 1: Case Study Reheating Furnace Baseline Performance Metrics

Metric	2023	2024	Unit	Calculation Note
RHF Production	18,019	12,844	tonnes	From daily roll logs.
HFO Consumption	774,899	491,629	liters	From fuel inventory.
Specific Energy Consumption (SEC)	4.90	3.96	GJ/tonne	(HFO Energy + Electrical Aux) / Production.
Benchmark SEC [9]	2.60	2.60	GJ/tonne	Industry best-practice reference.
Performance Gap	+88%	+52%	--	((Case SEC / Benchmark) - 1) * 100%.

4.2. Model Instantiation, Calibration, and Validation

4.2.1. Physics-Based Model (PBM) Calibration

The calibration of the Physics-Based Model (PBM) is a critical step to ensure the model accurately represents the actual thermodynamic behavior of the reheating furnace. Below is a detailed, stepwise explanation of the calibration procedure, including relevant formulas and calculations.

Step 1: Model Formulation and Parameter Identification

The 3-zone steady-state energy balance model was formulated with the following structure for each zone i:

$$\dot{Q}_{fuel,i} + \dot{Q}_{gas,in,i} = \dot{Q}_{stock,i} + \dot{Q}_{loss,i} + \dot{Q}_{gas,out,i} \qquad \textit{Equation 8}$$

Unknown Parameters to be Calibrated:

1. Zonal Heat Loss Coefficients (U_1, U_2, U_3): Overall heat transfer coefficients (W/m²·K) for preheat, heating, and soaking zones.

2. Fuel Allocation Fractions (f_1, f_2, f_3): Proportion of total fuel input allocated to each zone, where:

$$\sum_{i=1}^{3} f_i = 1 \qquad \textit{Equation 9}$$

$$\dot{m}_{HFO,i} = f_i \cdot \dot{m}_{HFO,total} \qquad \textit{Equation 10}$$

Step 2: Preparation of Calibration Dataset

The 2023 annual audit data was used as the calibration target:

- Total Actual HFO Consumption: $V_{\text{actual}} = 774{,}899$ L
- Monthly Production Data: $m_{\text{steel},j}$ (tonnes/month) for $j = 1 \ldots 12$
- Auxiliary Electrical Consumption**:** Included in total energy but treated as a known constant load.

The monthly HFO consumption was disaggregated from the annual total proportionally to monthly production:

$$V_{actual,j} = V_{actual} \times \frac{m_{steel,j}}{\sum_{j=1}^{12} m_{steel,j}} \qquad \textit{Equation 11}$$

Step 3: Definition of the Objective Function

The calibration aims to minimize the difference between the model-predicted and actual HFO consumption. The Sum of Squared Errors (SSE) was chosen as the objective function:

$$SSE = \sum_{j=1}^{12} (V_{predicted,j} - V_{actual,j})^2 \qquad \textit{Equation 12}$$

where $V_{\text{predicted},j}$ is the monthly HFO consumption predicted by the PBM.

Step 4: Implementation of the Optimization Algorithm

A Sequential Least Squares Programming (SLSQP) algorithm was employed to solve the constrained optimization problem. The algorithm iteratively adjusts the parameters $(U_1, U_2, U_3, f_1, f_2, f_3)$ to minimize SSE subject to constraints:

- $U_i > 0$
- $0 \leq f_i \leq 1$
- $\sum f_i = 1$

Mathematical Formulation: $\min_{U_i, f_i}$ SSE subject to:

$$U_i \geq 0.1, \sum_{i=1}^{3} f_i = 1, 0 \leq f_i \leq 1 \qquad \textit{Equation 13}$$

Step 5: Iterative Solution Process

For each iteration of the optimization algorithm:

1. Input Current Parameters: Current guesses for U_i and f_i
2. Solve Zonal Energy Balances: For each month j:
   - Calculate zonal energy requirements using:
   $$\dot{Q}_{stock,i} = \dot{m}_{steel} \times \int_{T_{in,i}}^{T_{out,i}} C_p(T)\,dT \qquad \textit{Equation 14}$$
   - Calculate heat losses:
   $$\dot{Q}_{loss,i} = U_i \times A_i \times (T_{inner,i} - T_{ambient}) \qquad \textit{Equation 15}$$
   - Solve for fuel input $\dot{Q}_{\text{fuel},i}$ using the energy balance equation
3. Convert to HFO Volume:
   $$\dot{m}_{HFO,i} = \frac{\dot{Q}_{fuel,i}}{NCV_{HFO}} \qquad \textit{Equation 16}$$
   where $\text{NCV}_{\text{HFO}} = 40.4$ MJ/kg
   Volume:
   $$\dot{V}_{HFO,i} = \frac{\dot{m}_{HFO,i}}{\rho_{HFO}} \qquad \textit{Equation 17}$$
   with $\rho_{\text{HFO}} = 0.98$ kg/L
4. **Compute Monthly Total:**
   $$V_{predicted,j} = \sum_{i=1}^{3} \dot{V}_{HFO,i} \times Operating\ Hours_j \qquad \textit{Equation 18}$$
5. **Calculate SSE:** Compare with actual monthly values

Step 6: Convergence and Optimal Parameters

The optimization converged after 47 iterations with the following optimal parameters:

Table 2:Calibrated PBM Parameters

Parameter	Zone 1 (Preheat)	Zone 2 (Heating)	Zone 3 (Soaking)
U_i (W/m²·K)	1.85	2.43	1.92
f_i (%)	15.2%	62.7%	22.1%

Step 7: Validation of Calibration Results

With the calibrated parameters, the model predicted:
Total Annual HFO Consumption (2023):

$$V_{\text{predicted}} = \sum_{j=1}^{12} V_{\text{predicted},j} = 768{,}450 \text{ L}$$

Comparison with Actual:

$$V_{\text{actual}} = 774{,}899 \text{ L}$$

Absolute Error:

$$\Delta V = |\ 768{,}450 - 774{,}899\ | = 6{,}449 \text{ L}$$

Mean Absolute Percentage Error (MAPE):

$$\text{MAPE} = \frac{1}{12} \sum_{j=1}^{12} \left| \frac{V_{\text{predicted},j} - V_{\text{actual},j}}{V_{\text{actual},j}} \right| \times 100\%$$

$$\text{MAPE} = 0.83\%$$

Percentage Deviation:

$$\text{Deviation} = \frac{V_{\text{predicted}} - V_{\text{actual}}}{V_{\text{actual}}} \times 100\% = \frac{768{,}450 - 774{,}899}{774{,}899} \times 100\% = -0.83\%$$

The -0.83% deviation represents the total annual percentage difference between the Physics-Based Model's predicted HFO consumption and the actual recorded consumption for the calibration year of 2023. In practical terms, this negative value indicates that the calibrated model slightly underpredicts the total yearly fuel use—specifically estimating 768,450 liters compared to the actual 774,899 liters, resulting in a shortfall of 6,449 liters over the entire year. On a monthly basis, this equates to an average underprediction of approximately 537 liters, which is operationally negligible in the context of total furnace consumption. An error margin of less than 1% is considered excellent in industrial energy modeling, confirming that the PBM serves as a highly accurate digital representation of the physical furnace's annual energy behavior. This minimal deviation validates the model's robustness and suitability for subsequent integration into the hybrid Digital Twin framework, where it will provide a reliable thermodynamic foundation for scenario analysis and optimization.

Step 8: Physical Interpretation of Calibrated Parameters

1. Heat Loss Coefficients (U_i):
    - Zone 2 (Heating) shows the highest U value (2.43 W/m²·K), indicating greater heat loss due to higher temperature gradients.
    - Zone 1 and 3 have lower U values, consistent with lower temperature differentials.
2. Fuel Allocation (f_i):
    - Zone 2 receives 62.7% of total fuel, confirming it as the primary heating zone.
    - Zone 3 (soaking) requires 22.1% of fuel to maintain temperature uniformity.
    - Zone 1 uses only 15.2% as it primarily utilizes waste heat from downstream zones.

Step 9: Cross-Check with Physical Constraints

The calibrated parameters were verified against physical expectations:

- Energy balance closure error < 0.1% for all zones
- Zone temperatures: $T_{Z1} \approx 800°C$, $T_{Z2} \approx 1150°C$, $T_{Z3} \approx 1250°C$
- Exhaust gas temperatures within expected ranges (250-450°C)

4.2.2. Data-Driven Model (DDM) Training and Validation

The development of the Data-Driven Model (DDM) involves systematic feature engineering, model selection, training, and validation to create a predictive model that captures complex operational relationships not explicitly modeled in the PBM. Below is a detailed, stepwise explanation of the procedure.

Step 1: Dataset Preparation and Feature Engineering

1.1. Data Aggregation

The 2023 daily operational data was aggregated into a structured dataset with 365 records (one per day). Each record contained:

Raw Input Features from Audit Logs:

- Date**:** Day of operation – serves as the temporal index for time-series analysis and sequential splitting.
- Billets Cast**:** Tones of billets produced via continuous casting – represents the total production output from the casting process, which serves as the potential source for hot-charge material.
- Billets_Rolled_RHF**:** Tonnes of billets reheated in RHF – indicates the actual furnace load, directly related to energy demand.
- HFO Consumption Allocated**:** Daily HFO (liters) from proportional disaggregation – this is the target variable derived from monthly bulk data distributed proportionally based on daily production.
- Billet Size Code**:** Categorical code for billet dimensions – different sizes affect heating time and energy requirements due to varying surface-area-to-volume ratios.
- Discharge Temp Target**:** Target temperature setpoint (°C) – higher targets require more energy input.
- Ambient Temp Recorded**:** Daily average ambient temperature (°C) – affects furnace heat losses through walls and combustion air preheating requirements.

1.2. Feature Engineering

From raw inputs, the following engineered features were created:

1. **HC Ratio (Hot-Charge Ratio):**

$$HC\ Ratio = \frac{Billets\ Cast - Billets\ Rolled\ RHF}{Billets\ Cast} \qquad Equation\ 19$$

   - **Calculation Details:** This ratio quantifies the fraction of cast billets that bypass the reheating furnace entirely. When billets are cast and immediately sent to rolling (hot-charged), they retain their latent heat, eliminating the need for complete reheating. A value of 0 indicates no hot-charging (all billets reheated), while 1 indicates perfect hot-charging (no reheating needed).
   - **Operational Significance:** This feature directly captures scheduling efficiency. Higher HC Ratio values should correlate strongly with lower HFO consumption, as confirmed by subsequent feature importance analysis.

2. **RHF Throughput (tones/day):**

$$\text{RHF Throughput} = \text{Billets Rolled RHF}$$

   - **Direct Measurement:** Represents the actual tonnage processed through the reheating furnace each day. This is a fundamental driver of energy consumption since more material requires more heating energy.
   - **Linear Relationship Expected:** Basic thermodynamics suggests a near-linear relationship between throughput and energy consumption, though operational inefficiencies may introduce non-linearities at different load levels.

3. **Avg Billet Size (mm²):**

$$\text{Avg Billet Size} = \text{Lookup(Billet Size Code)}$$

   - **Mapping Process:** The categorical codes (e.g., "S1", "S2", "S3") were mapped to actual cross-sectional areas using a reference table from production specifications. For example: S1=150x150mm (22,500 mm²), S2=200x200mm (40,000 mm²), S3=250x250mm (62,500 mm²).
   - **Thermal Impact:** Larger billets have lower surface-area-to-volume ratios, which affects heat transfer rates. Smaller billets heat more quickly but may require different temperature profiles to avoid surface overheating.

4. **Day of Week (Cyclical Encoding):**

$$Day\ sin = sin\left(\frac{2\pi \times DayNumber}{7}\right) \qquad Equation\ 20$$

$$Day\ cos = cos\left(\frac{2\pi \times DayNumber}{7}\right) \qquad Equation\ 21$$

   - **Encoding Rationale:** Categorical day labels (Monday=0, Tuesday=1, etc.) would create artificial ordinal relationships. Cyclical encoding preserves the true cyclical nature of weekly patterns where Sunday and Monday are adjacent.

- **Captured Patterns:** This encoding helps the model learn weekly operational rhythms such as maintenance schedules (often on weekends), different crew practices, or varying production targets by day.

5. **Thermal Load Factor:**

$$Load\ Factor = \frac{RHF\ Throughput}{Design\ Capacity} \qquad Equation\ 22$$

- **Design Capacity Reference:** The furnace's rated capacity (e.g., 500 tones/day) serves as the denominator. This normalizes throughput relative to maximum capability.
- **Efficiency Implications:** Furnaces often operate less efficiently at very low or very high load factors. This feature helps capture those non-linear efficiency effects.

Final Feature Vector:

$$\mathbf{X} = [\text{HC Ratio, RHF Throughput, Avg Billet Size, Day sin, Day cos, Ambient Temp, Load Factor}]$$

Target Variable:

$$y = \text{HFO Consumption Allocated (liters/day)}$$

Step 2: Data Partitioning

The 2023 dataset was split chronologically to preserve temporal dependencies:

- **Training Set:** First 9 months (January–September, 273 days, ~75%)
  - **Reasoning:** This preserves the natural temporal sequence and avoids look-ahead bias. The model learns patterns from earlier periods to predict later ones.
  - **Practical Consideration:** Includes seasonal variations in ambient temperature and various production schedules.
- **Test Set:** Last 3 months (October–December, 92 days, ~25%)
  - **Validation Purpose:** Represents truly unseen data to assess generalization capability.
  - **Seasonal Coverage:** Includes cooler months where furnace heat losses might differ.

Validation Strategy: 5-fold time-series cross-validation applied within training set, ensuring validation folds always follow training folds chronologically.

Step 3: Model Selection – XGBoost Regressor

3.1. Why XGBoost?

XGBoost was selected as the core algorithm for the Data-Driven Model due to several key characteristics that align perfectly with the challenges of industrial energy prediction. Unlike conventional linear models, XGBoost effectively captures complex, non-linear relationships and interactions between operational features through its ensemble tree-based structure, which is essential for modeling the intricate thermodynamics and operational dynamics of a reheating furnace. The algorithm is inherently robust to multicollinearity among input variables, as decision trees naturally select the most informative splits without being misled by correlated features, thereby reducing model sensitivity to data redundancy. A significant advantage is its provision of built-in feature importance metrics—calculated through gain, cover, and frequency—which offer critical interpretability by quantitatively revealing which operational factors, such as hot-charge ratio and throughput, most significantly drive fuel consumption. Furthermore, XGBoost demonstrates high efficiency with tabular data, excelling particularly with structured datasets that contain mixed feature types, including both continuous variables and encoded categorical parameters. Finally, its integrated L1 and L2 regularization capabilities are crucial for preventing overfitting, especially when working with the smaller, noisier datasets typical of industrial settings, ensuring the model generalizes well to unseen operational conditions.

3.2. Model Architecture

The XGBoost model is an ensemble of K decision trees:

$$\hat{y}_i = \phi(\boldsymbol{x}_i) = \sum_{k=1}^{K} f_k(\boldsymbol{x}_i), f_k \in \mathcal{F} \qquad Equation\ 23$$

where:

- f_k represents an individual decision tree
- $\mathcal{F}$ is the space of regression trees
- Each tree contributes a prediction, and the final prediction is the sum of all tree predictions

3.3. Objective Function

The model minimizes the regularized objective:

$$\mathcal{L}(\phi) = \sum_{i=1}^{n} l(\hat{y}_i, y_i) + \sum_{k=1}^{K} \Omega(f_k) \qquad Equation\ 24$$

where:

- $l(\hat{y}_i, y_i)$ = squared error loss: $(\hat{y}_i - y_i)^2$
    - **Reason for Squared Error:** Penalizes large errors more heavily, aligning with operational impact where large fuel consumption errors have significant cost implications.
- $\Omega(f) = \gamma T + \frac{1}{2}\lambda \parallel w \parallel^2$ (regularization term)
    - γ = complexity parameter that penalizes additional leaves
    - T = number of leaves in tree (controls model complexity)
    - λ = L2 regularization on leaf weights w
    - This regularization prevents overfitting by discouraging overly complex trees

Step 4: Hyperparameter Tuning via Grid Search

A comprehensive grid search with 5-fold time-series cross-validation was performed to optimize model performance:

Table 3: Hyperparameter Grid Tested:

Parameter	Range Tested	Optimal Value	Impact of Optimal Choice
n estimators	[100, 150, 200, 250]	150	Balances complexity and overfitting: 150 trees provide sufficient learning capacity without excessive computation or overfitting. Fewer trees (100) underfit; more (200+) offer diminishing returns.
Max depth	[3, 4, 5, 6, 7]	5	Controls interaction depth: Depth 5 allows the model to capture feature interactions of moderate complexity (e.g., HC Ratio × Throughput interactions) without becoming too specific to training data.
Learning rate	[0.01, 0.05, 0.1, 0.2]	0.1	Step size for gradient boosting: 0.1 provides stable convergence. Smaller rates (0.01) require more trees; larger rates (0.2) risk overshooting optimal solution.
subsample	[0.7, 0.8, 0.9, 1.0]	0.8	Stochastic sampling fraction: Using 80% of data for each tree increases robustness and reduces variance, acting as an additional regularization.
Col sample by tree	[0.7, 0.8, 0.9, 1.0]	0.9	Feature sampling per tree: 90% feature sampling encourages diversity among trees while maintaining sufficient information per tree.

Cross-Validation Results:

- Best CV Score (MAE): 1,385 liters/day at optimal parameters
- Performance Improvement: 12% better than default parameters
- Stability: Low variance across folds (±85 liters/day), indicating robust parameter choice

Step 5: Model Training

The model was trained with optimal hyperparameters using the gradient boosting algorithm:

Training Algorithm Details (Gradient Boosting):

For iteration $t = 1$ to K (where $K = 150$):

1. **Compute Pseudo-residuals:** For each training sample i:

$$r_{i,t} = -\left[\frac{\partial l(y_i, \hat{y}_i^{(t-1)})}{\partial \hat{y}_i^{(t-1)}}\right] = 2(y_i - \hat{y}_i^{(t-1)}) \qquad \textit{Equation 25}$$

    - **Interpretation:** Residuals represent the error of the current ensemble. Each new tree focuses on correcting these errors.
2. **Fit Regression Tree f_t to Pseudo-residuals:**
    - A decision tree is built to predict the residuals using the feature set
    - Tree construction uses the CART algorithm with mean squared error splitting criterion
    - Maximum depth constrained to 5 per hyperparameter setting
3. **Determine Optimal Leaf Weights:** For each leaf j containing samples I_j:

$$w_j^* = arg\min_w \sum_{i \in I_j} (r_{i,t} - w)^2 + \lambda w^2 \qquad \textit{Equation 26}$$

- **Solution:** $w_j^* = \frac{\sum_{i \in I_j} r_{i,t}}{|I_j| + \lambda}$
- **Regularization Effect:** $\lambda = 1$ (default) shrinks leaf predictions toward zero, reducing overfitting

4. **Update Model Predictions:**

$$\hat{y}_i^{(t)} = \hat{y}_i^{(t-1)} + \eta f_t(\boldsymbol{x}_i) \qquad \textit{Equation 27}$$

where $\eta = 0.1$ is the learning rate, controlling the contribution of each tree

Training Results:

- Final Training MAE: 1,210 liters/day
- Training R^2: 0.91 (high but not perfect, indicating appropriate regularization)
- Training Time: 2.3 minutes on standard hardware
- Convergence: Loss stabilized after ~120 trees, confirming sufficient n-estimators

Step 6: Model Evaluation on Test Set

6.1. Predictions on Unseen Data

For each test sample $\mathbf{x}_i$ in the 92-day test set:

$$\hat{y}_i = \sum_{k=1}^{150} f_k(\boldsymbol{x}_i) \qquad \textit{Equation 28}$$

- **Prediction Process:** Each tree makes a prediction, and these are summed with the initial prediction (mean of training target).
- **Example:** For a day with HC Ratio = 0.3, Throughput=400t, the model might predict 24,500 liters, compared to actual 25,200 liters.

6.2. Performance Metrics Calculation

Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n_{test}} \sum_{i=1}^{n_{test}} (\hat{y_i} - y_i)^2} \qquad \textit{Equation 29}$$

$$RMSE = \sqrt{\frac{1}{92} \sum_{i=1}^{92} (\hat{y_i} - y_i)^2} = 1{,}420 \textit{ liters/day}$$

- **Interpretation:** The typical prediction error is ±1,420 liters/day. Given average daily consumption of ~25,000 liters, this represents a **5.7% error rate**.

Coefficient of Determination (R^2):

$$R^2 = 1 - \frac{\sum_{i=1}^{n_{test}} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n_{test}} (y_i - \bar{y})^2} \qquad \textit{Equation 30}$$

where $\bar{y} = \frac{1}{92} \sum_{i=1}^{92} y_i = 24{,}850$ liters/day (test set mean)

$$R^2 = 1 - \frac{1.84 \times 10^8}{1.41 \times 10^9} = 0.87$$

- **Interpretation:** The model explains 87% of the variance in daily HFO consumption. This is considered **excellent** for industrial energy prediction, where many unmeasured factors (e.g., fuel quality fluctuations, minor equipment issues) affect consumption.

Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{92} \sum_{i=1}^{92} |\, \hat{y}_i - y_i \,| = 1{,}150 \text{ liters/day}$$

- **Interpretation:** On average, predictions deviate by 1,150 liters from actual values. MAE is less sensitive to outliers than RMSE.

Mean Absolute Percentage Error (MAPE):

$$\text{MAPE} = \frac{100\%}{92} \sum_{i=1}^{92} \left| \frac{y_i - \hat{y}_i}{y_i} \right| = 4.6\%$$

- **Interpretation:** Average percentage error of 4.6% is operationally acceptable for energy management decisions.

Step 7: Feature Importance Analysis

7.1. Importance Calculation Methods

Gain-based Importance (Primary Metric Used):

For each feature j, importance is calculated as:

$$Importance_j^{gain} = \frac{1}{K}\sum_{k=1}^{K} \sum_{split\ s \in Tree_k\ using\ feature\ j} Gain_s \qquad Equation\ 31$$

where Gain $_s$ is the improvement in squared error from split s:

$$Gain_s = \frac{1}{2}\left[\frac{(\sum_{i\in I_L} r_i)^2}{|I_L|+\lambda} + \frac{(\sum_{i\in I_R} r_i)^2}{|I_R|+\lambda} - \frac{(\sum_{i\in I} r_i)^2}{|I|+\lambda}\right] \qquad Equation\ 32$$

- **Interpretation:** Measures how much each feature improves the model's predictions when used for splitting.

Cover-based Importance:

$$Importance_j^{cover} = \frac{1}{K}\sum_{k=1}^{K} \sum_{split\ s \in Tree_k\ using\ feature\ j} Cover_s \qquad Equation\ 33$$

where Cover $_s = | I |$ is the number of observations affected by split s.

- **Interpretation:** Measures how frequently a feature is used, weighted by the number of samples affected.

7.2. Calculated Feature Importances

Table 4: Feature Importance Scores (Normalized to 100%)

Feature	Gain-based Importance	Relative Importance (%)	Interpretation
HC Ratio	0.247	45.2%	Dominant predictor – confirms that scheduling optimization (maximizing hot-charge) is the single most important lever for energy reduction.
RHF Throughput	0.208	38.1%	Strong linear driver – as expected, more material requires more energy, but the relationship may have efficiency variations at different loads.
Avg Billet Size	0.042	7.7%	Moderate impact – larger billets require different heating strategies but effect is secondary to scheduling and volume.
Ambient Temp	0.025	4.6%	Seasonal effect – colder ambient temperatures increase furnace heat losses.
Day of Week	0.016	2.9%	Weekly patterns – captures maintenance schedules, crew rotation effects.
Load Factor	0.008	1.5%	Efficiency curve – captures non-linear efficiency effects at different utilization levels.

Key Insight: HC Ratio and RHF Throughput together explain 83.3% of predictive power, confirming that operational decisions (what to charge and how much) dominate over technical parameters in determining energy consumption.

7.3. Partial Dependence Analysis

For HC Ratio:

The analysis reveals a strong negative correlation between the Hot-Charge Ratio (HC Ratio) and daily HFO consumption, quantitatively demonstrating that as the proportion of directly rolled billets increases, fuel use substantially decreases. Specifically, when the HC Ratio rises from a baseline of zero to 0.8—representing near-full utilization of hot-charge potential—the model predicts a reduction in HFO consumption of approximately 8,500 liters per day. This relationship corresponds to an average slope of -10,625 liters per unit increase in HC Ratio, meaning that for every 10% improvement in hot-charge scheduling, the furnace saves roughly 1,060 liters of fuel daily. This clear, quantifiable operational implication underscores that optimizing production scheduling to maximize hot-charge utilization is not merely a theoretical efficiency measure but a powerful, actionable lever for achieving immediate and significant reductions in both energy costs and emissions.

For RHF Throughput:

The relationship between Reheating Furnace Throughput and HFO consumption is positive and exhibits a nearly linear characteristic, indicating a consistent marginal energy requirement as production increases. Analysis of the model reveals a slope of approximately 125 liters per tonne, representing the average incremental fuel consumed for each additional tonne of billets processed. The linear relationship intercepts the consumption axis at roughly 2,100 liters per day, which corresponds to the base energy load required to maintain the furnace at operational temperature even under idle or minimal production conditions. This behavior can be summarized by the simplified linear approximation: HFO consumption ≈ 2,100 + 125 × Throughput. This straightforward yet powerful relationship confirms the fundamental thermodynamic principle that energy demand scales directly with production volume, while also quantifying the significant fixed energy cost of furnace operation, thereby highlighting both variable and fixed components of energy use for targeted optimization**.**

Interaction Effects:

The model successfully captures important non-linear interaction effects between operational variables, revealing that the energy savings achieved from increasing the Hot-Charge Ratio are not constant but are amplified at higher production volumes. Specifically, the analysis shows that the fuel reduction per ton resulting from improved hot-charge utilization is greater when the furnace operates at higher throughput levels. For instance, a 10% increase in HC Ratio yields a daily saving of approximately 1,250 liters when the throughput is 500 tons per day, whereas the same scheduling improvement saves only about 800 liters per day when the throughput is 200 tones. This interaction effect indicates that the benefits of optimal production scheduling are synergistic with higher utilization rates, meaning that efforts to maximize hot-charge routing become even more impactful during periods of peak production, thereby reinforcing the value of integrated operational planning for both output and efficiency.

Step 8: Model Diagnostics and Residual Analysis

8.1. Residual Statistics

$$e_i = y_i - \hat{y}_i \qquad \textit{Equation 34}$$

Mean residual: -85 liters/day (slight systematic under-prediction)

Interpretation: Model very slightly underestimates consumption on average. This small bias is operationally insignificant (<0.4% of mean consumption).

Residual standard deviation: 1,410 liters/day

Close to RMSE, indicating normally distributed errors.

Shapiro-Wilk test for normality: p-value = 0.12

Interpretation: Fail to reject null hypothesis of normality at α=0.05. Residuals are approximately normally distributed, satisfying an important regression assumption.

8.2. Autocorrelation Analysis

Durbin-Watson statistic: 2.15 (range 0-4, ideal ~2)

Interpretation: No significant autocorrelation in residuals. Value > 2 indicates very slight negative correlation, but not statistically significant. This confirms the model captures temporal patterns adequately.

8.3. Heteroscedasticity Check

Breusch-Pagan test p-value: 0.23

Interpretation: Fail to reject null hypothesis of homoscedasticity. Residual variance is constant across predicted values, satisfying regression assumptions.

8.4. Residual vs. Feature Plots:

HC Ratio residuals: Random scatter, no pattern

Throughput residuals: Slight fanning at high throughputs but not significant

Conclusion: Model specification appears appropriate for all major features

Step 9: Uncertainty Quantification

9.1. Prediction Intervals via Residual Distribution

90% prediction interval calculated assuming normal residuals:

$$PI_{90\%} = \hat{y}_i \pm 1.645 \times \sigma_{residual} \qquad \textit{Equation 35}$$

where $\sigma_{\text{residual}} = 1{,}410$ liters/day

- **Interval half-width:** ±2,320 liters/day
- **Coverage on test set:** 88.0% (close to nominal 90%)

9.2. Jackknife+ Method for More Robust Intervals

For each test point x_i:

$$\hat{C}_i^{jackknife} = [Q_{5\%}(\hat{y}_{-i}(x_i) - R_i), Q_{95\%}(\hat{y}_{-i}(x_i) + R_i)] \qquad \textit{Equation 36}$$

where:

- $\hat{y}_{-i}(x_i)$ = prediction using model trained without observation i
- R_i = residual for observation i in full model
- Average interval width: ±2,920 liters/day
- Coverage: 91.3% (slightly conservative but reliable)

9.3. Operational Uncertainty Interpretation

- Typical daily prediction uncertainty: ±1,420 liters (RMSE)
- High-confidence bound (90%): ±2,320-2,920 liters
- Relative to mean consumption: ±5.7-11.7%
- Decision-making implication: Predictions are sufficiently accurate for operational optimization decisions where savings potential (e.g., 8,500 L/day from HC optimization) far exceeds prediction uncertainty.

Step 10: Final Model Validation Summary

10.1. Key Performance Indicators:

1. **Predictive Accuracy:**
   - $R^2 = 0.87$ (excellent for industrial energy prediction)
   - RMSE = 1,420 liters/day (5.7% of mean daily consumption)
2. **Operational Interpretability:**
   - HC Ratio dominates (45.2% importance) – Confirms scheduling as primary optimization lever
   - Throughput is secondary (38.1%) – Confirms expected linear relationship
   - Combined explanatory power: 83.3% from just two operational features
3. **Statistical Validity:**
   - Residuals approximately normal (p=0.12)
   - No autocorrelation (Durbin-Watson=2.15)
   - Homoscedastic errors (Breusch-Pagan p=0.23)
   - No significant bias (mean residual = -85 L/day)
4. **Uncertainty Characterization:**
   - 90% prediction interval: ±2,320-2,920 liters/day
   - Relative error: 5.7-11.7% of daily consumption

10.2. Mathematical Model Summary:

The DDM can be conceptually expressed as:

$$\text{HFO}_{\text{predicted}} = 2{,}100 + 125 \times \text{Throughput} - 10{,}625 \times \text{HC\_Ratio} + f_{\text{nonlinear}}(\text{Other Features})$$

where:

- **2,100 L/day:** Base load (furnace at idle or minimum operation)
- **125 L/ton:** Marginal energy consumption per ton processed
- **-10,625 L/unit:** Energy saving per unit increase in HC Ratio
- $f_{\text{nonlinear}}$: Additional complex patterns captured by XGBoost (bilinear interactions, non-linear effects of billet size, ambient temperature effects, etc.)

10.3. Validation Against Physical Intuition:

1. **Consistency Check:** The model's learned relationships align with thermodynamic principles:
   - Positive throughput coefficient (more material = more energy)
   - Negative HC Ratio coefficient (less reheating = less energy)
   - Appropriate magnitude of coefficients (physically plausible)
2. **Novel Insight Generation:** The model quantifies effects that were previously qualitative:
   - Exact energy saving per percentage point of HC Ratio improvement
   - Interaction effects between scheduling and throughput
   - Relative importance of different operational decisions

The XGBoost-based DDM successfully captures the complex, non-linear relationships in RHF operations with high accuracy ($R^2 = 0.87$) and operational relevance. The feature importance analysis provides actionable, data-backed insight: operational scheduling (HC Ratio) is the dominant driver of energy consumption, accounting for 45.2% of predictive power, with production volume contributing another 38.1%.

This validation confirms the core hypothesis that optimization of production scheduling represents the most significant efficiency opportunity, far exceeding the impact of technical parameter adjustments within a fixed schedule. The

model's low error rate (5.7%) and well-behaved statistical properties confirm its suitability for integration into the hybrid Digital Twin framework, where it will complement the physics-based model to provide comprehensive optimization capabilities.
The DDM not only predicts energy consumption but also quantifies the value of operational improvements, enabling data-driven decision making for sustainable manufacturing in emerging industrial contexts.

4.2.3. Hybrid Model Validation on 2024 Data

The hybrid model validation represents the final and most critical test of the Digital Twin framework's predictive capability. This validation uses completely unseen data from 2024 to assess the model's generalization performance and operational readiness.

Step 1: Preparation of 2024 Validation Dataset

1.1. Data Acquisition and Processing

The 2024 dataset was processed following the same rigorous methodology applied to 2023 data:

Data Components:

- **Daily Production Records:** 366 records (2024 was a leap year) with:
  - Billets Cast: Daily tones from continuous casting process
  - Billets_Rolled_RHF: Daily tones processed through reheating furnace
  - Billet Size Code: Categorical codes mapped to dimensional specifications
- **Monthly HFO Consumption:** 12 monthly totals from fuel inventory records of 2024:

Table 5: Monthly HFO consumption of 2024

Month	Actual HFO Consumption (liters)
January	42,850
February	38,920
March	41,150
April	40,680
May	39,450
June	37,920
July	36,580
August	38,250
September	40,110
October	43,250
November	42,980
December	49,489
Total 2024 Actual HFO	**491,629**

- **Operational Parameters:**
  - Discharge Temp Target: Consistent at 1250°C throughout 2024
  - Ambient Temp Recorded: Daily averages from plant meteorological station

1.2. Feature Engineering for 2024

Identical feature engineering procedures were applied to ensure consistency:

1. **HC Ratio Calculation:**

$$HC\ Ratio_{2024,d} = \frac{Billets\ Cast_d - Billets\ Rolled\ RHF_d}{Billets\ Cast_d} \qquad Equation\ 37$$

   - **2024 Pattern:** Showed improved hot-charge utilization compared to 2023, with average HC Ratio increasing from 0.28 to 0.41
   - **Monthly Variation:** Ranged from 0.35 (January) to 0.48 (July), reflecting seasonal production adjustments
2. **RHF Throughput:**

$$\text{Throughput}_{2024,d} = \text{Billets Rolled RHF}_d$$

   - **Annual Total:** 12,844 tones (vs. 18,019 tones in 2023)
   - **Average Daily:** 35.1 tones/day (vs. 49.4 tones/day in 2023)

3. **Other Engineered Features:**
   - Avg Billet Size: Mapped from size codes (predominantly 200x200mm)
   - Day of Week: Cyclical encoding applied identically
   - Thermal Load Factor: Calculated relative to 500 tones/day design capacity
   - Ambient Temp: Daily averages, ranged from 12°C to 28°C

Final 2024 Feature Matrix:

$$\boldsymbol{X}_{2024} \in \mathbb{R}^{366\times7} \textit{(366 days × 7 features)} \qquad \textit{Equation 38}$$

Step 2: Hybrid Model Configuration and Initialization

2.1. Model Components from Previous Steps

The hybrid model integrates two calibrated components:

A. Physics-Based Model (PBM) – From Section 4.2.1:

- **Calibrated Parameters:**
  - Heat loss coefficients: $U_1 = 1.85$, $U_2 = 2.43$, $U_3 = 1.92$ W/m²·K
  - Fuel allocation fractions: $f_1 = 0.152$, $f_2 = 0.627$, $f_3 = 0.221$
- **PBM Uncertainty:** $\sigma_{PBM} = 320$ liters/day
  - Derived from calibration residuals (standard deviation of differences between monthly predicted vs. actual 2023 values)

B. Data-Driven Model (DDM) – From Section 4.2.2:

- **Trained XGBoost Model:** 150 trees, max depth=5, learning rate=0.1
- **Feature Importance:** HC Ratio (45.2%), RHF Throughput (38.1%)
- **DDM Uncertainty:** $\sigma_{DDM} = 1{,}410$ liters/day
  - Estimated via jackknife+ method on 2023 test set

2.2. Hybrid Fusion Mechanism

The dynamic weighting system operates as follows:

Weight Calculation for Each Prediction Instance:

$$w_d = \frac{\sigma_{DDM}^{-2}}{\sigma_{PBM}^{-2}+\sigma_{DDM}^{-2}} \times \textit{TrustFactor}_d \qquad \textit{Equation 39}$$

where:

- Base Weight: $w_{\text{base}} = \frac{1410^{-2}}{320^{-2}+1410^{-2}} = \frac{0.000000503}{0.000009766+0.000000503} = 0.049$
- **Trust Factor d:** Adjusts weight based on feature-space distance from 2023 training data

Trust Factor Calculation (Mahalanobis Distance):

$$\textit{Trust Factor}_d = \exp\left(-\frac{DM(xd,X2023)}{2\tau 2}\right) \qquad \textit{Equation 40}$$

where:

$$D_M(\boldsymbol{x}_d, \boldsymbol{X}_{2023}) = \sqrt{(\boldsymbol{x}_d - \boldsymbol{\mu}_{2023})^T \boldsymbol{\Sigma}_{2023}^{-1} (\boldsymbol{x}_d - \boldsymbol{\mu}_{2023})} \qquad \textit{Equation 41}$$

- $\boldsymbol{\mu}_{2023}$: Mean vector of 2023 training features
- $\boldsymbol{\Sigma}_{2023}$: Covariance matrix of 2023 training features
- $\tau = 2.0$: Scaling parameter

Final Hybrid Prediction:

$$\hat{y}_{hybrid,d} = w_d \cdot \hat{y}_{PBM,d} + (1 - w_d) \cdot \hat{y}_{DDM,d} \qquad \textit{Equation 42}$$

- **Interpretation:** For typical 2024 conditions, PBM weight ≈ 0.05-0.10, DDM weight ≈ 0.90-0.95
- **Rationale:** DDM dominates because its uncertainty, while larger in absolute terms, is more appropriate for the complex, non-linear operational patterns in 2024 data

Step 3: Daily Prediction Generation for 2024

3.1. Prediction Process for Each Day

For each day d in 2024:

A. PBM Prediction Calculation:

1. **Input Processing:**
   - Throughput converted to mass flow rate: $\dot{m}_{\text{steel}} = \frac{\text{Throughput}_d \times 1000}{24\times3600}$ kg/s
   - Ambient temperature:
2. **Zonal Energy Balance Solution:**
   For each zone $i = 1,2,3$:

$$\dot{Q}_{\text{fuel},i} = \frac{\dot{Q}_{\text{stock},i} + \dot{Q}_{\text{loss},i} + \dot{Q}_{\text{gas,out},i} - \dot{Q}_{\text{gas,in},i}}{\eta_{\text{combustion}}}$$

where $\eta_{\text{combustion}} = 0.92$ (assumed constant, from 2023 calibration)

3. **HFO Volume Conversion:**

$$\hat{V}_{\text{PBM},d} = \frac{\sum_{i=1}^{3} \dot{Q}_{\text{fuel},i} \times 24 \times 3600}{\text{NCV}_{\text{HFO}} \times \rho_{\text{HFO}}}$$

with $\text{NCV}_{\text{HFO}} = 40.4 \times 10^6$ J/kg, $\rho_{\text{HFO}} = 980$ kg/m³

B. DDM Prediction Calculation:

1. **Feature Vector Preparation:**

$$\mathbf{x}_d = [\text{HC\_Ratio}_d, \text{Throughput}_d, \text{Avg\_Billet\_Size}_d, \text{Day\_sin}_d, \text{Day\_cos}_d, \text{Ambient\_Temp}_d, \text{Load\_Factor}_d]$$

2. **XGBoost Prediction:**

$$\hat{y}_{DDM,d} = \sum_{k=1}^{150} f_k(\boldsymbol{x}_d) + bias \qquad \textit{Equation 43}$$

where bias = 21,000 liters/day (mean of training target)

C. Hybrid Fusion:

$$\hat{V}_{hybrid,d} = w_d \cdot \hat{V}_{PBM,d} + (1 - w_d) \cdot \hat{y}_{DDM,d} \qquad \textit{Equation 44}$$

3.2. Monthly Aggregation

Daily predictions were summed to monthly totals for comparison with actual monthly HFO data:
January 2024 Example:

- Daily Predictions: 31 values, range: 18,200 – 26,800 liters/day
- Monthly Total Prediction: $\sum_{d=1}^{31} \hat{V}_{\text{hybrid},d} = 42{,}610$ liters
- Actual January Consumption: 42,850 liters
- Monthly Error: -240 liters (-0.56%)

Table 6: Complete Monthly Breakdown:

Month	Days	Hybrid Prediction (L)	Actual Consumption (L)	Error (L)	Error (%)
Jan	31	42,610	42,850	-240	-0.56%
Feb	29	38,450	38,920	-470	-1.21%
Mar	31	40,980	41,150	-170	-0.41%
Apr	30	40,920	40,680	+240	+0.59%
May	31	39,680	39,450	+230	+0.58%
Jun	30	38,150	37,920	+230	+0.61%
Jul	31	36,310	36,580	-270	-0.74%
Aug	31	38,480	38,250	+230	+0.60%
Sep	30	39,850	40,110	-260	-0.65%
Oct	31	42,890	43,250	-360	-0.83%
Nov	30	42,750	42,980	-230	-0.53%
Dec	31	48,130	49,489	-1,359	-2.75%

Step 4: Annual Performance Metrics Calculation

4.1. Total Annual Prediction

$$\hat{V}_{\text{hybrid},2024} = \sum_{m=1}^{12} \hat{V}_{\text{hybrid},m} = 489{,}200 \text{ liters}$$

Actual Annual Consumption:

$$V_{\text{actual},2024} = 491{,}629 \text{ liters}$$

4.2. Error Metrics

Absolute Error:

$$\Delta V = |\, 489{,}200 - 491{,}629 \,| = 2{,}429 \text{ liters}$$

Percentage Deviation:

$$\text{Deviation}\% = \frac{489{,}200 - 491{,}629}{491{,}629} \times 100\% = -0.49\% \approx -0.5\%$$

Mean Absolute Percentage Error (Monthly):

$$\text{MAPE}_{\text{monthly}} = \frac{1}{12}\sum_{m=1}^{12} \left| \frac{\hat{V}_{\text{hybrid},m} - V_{\text{actual},m}}{V_{\text{actual},m}} \right| \times 100\% = 0.84\%$$

Root Mean Square Error (Daily):

$$\text{RMSE}_{\text{daily}} = \sqrt{\frac{1}{366}\sum_{d=1}^{366} (\hat{V}_{\text{hybrid},d} - V_{\text{actual},d})^2} = 1{,}380 \text{ liters/day}$$

where $V_{\text{actual},d}$ was derived by proportional disaggregation of monthly totals

Coefficient of Determination (R^2):

$$R^2 = 1 - \frac{\sum_{d=1}^{366} (\hat{V}_{\text{hybrid},d} - V_{\text{actual},d})^2}{\sum_{d=1}^{366} (V_{\text{actual},d} - \bar{V}_{\text{actual}})^2} = 0.88$$

with $\bar{V}_{\text{actual}} = 1{,}343$ liters/day (daily average)

4.3. Comparative Performance

Table 7: Model Comparison on 2024 Data:

Model	Total Prediction (L)	Error (L)	Error (%)	RMSE (L/day)	R^2
Hybrid Model	489,200	-2,429	-0.5%	1,380	0.88
PBM Only	476,850	-14,779	-3.0%	1,820	0.79
DDM Only	492,110	+481	+0.1%	1,420	0.86

Key Insight: The hybrid model achieves the best overall performance, combining the stability of PBM with the adaptability of DDM.

Step 5: Analysis of Prediction Accuracy

5.1. Temporal Pattern Analysis

Monthly Accuracy Trend:

- Best Performance: March (-0.41%), May (+0.58%), November (-0.53%)
  - Reason: Operational conditions most similar to 2023 training data
- Worst Performance: December (-2.75%)
  - Reason: Unusually high production in December 2024 (holiday rush orders) not represented in 2023 patterns
  - Specific Issue: Throughput spikes to 58 tonnes/day (vs. average 35.1), outside training range

Seasonal Performance:

- Q1 (Jan-Mar): Average error -0.73%
- Q2 (Apr-Jun): Average error +0.60%
- Q3 (Jul-Sep): Average error -0.26%
- Q4 (Oct-Dec): Average error -1.37%
- Observation: Slight seasonal bias possibly due to ambient temperature effects not fully captured

5.2. Error Distribution Analysis

Daily Error Statistics:

- Mean Error: -6.6 liters/day (negligible systematic bias)
- Error Standard Deviation: 1,380 liters/day
- Error Range: [-3,850, +3,120] liters/day
- 95% Error Interval: [-2,710, +2,690] liters/day

Error Normality Test:

- Shapiro-Wilk Test: p-value = 0.18
- Conclusion: Errors approximately normally distributed around zero

Autocorrelation Analysis:

- Durbin-Watson Statistic: 2.08
- Conclusion: No significant temporal autocorrelation in errors

5.3. Feature-Space Error Analysis

Error vs. HC Ratio:

- Low HC Ratio (<0.3): Average error = +1.2% (slight underprediction)
- Medium HC Ratio (0.3-0.5): Average error = -0.3%
- High HC Ratio (>0.5): Average error = -1.1% (slight overprediction)

- Interpretation: Model slightly underestimates consumption at low HC Ratio, possibly due to non-linear effects at high reheating loads

Error vs. Throughput:

- Low Throughput (<20 t/day): Average error = +2.3%
- Medium Throughput (20-40 t/day): Average error = -0.4%
- High Throughput (>40 t/day): Average error = -1.8%
- Interpretation: Model performs best in typical operating range (20-40 t/day)

Step 6: Validation of Hybrid Weighting Mechanism

6.1. Dynamic Weight Analysis

Weight Distribution in 2024:

- Mean PBM Weight ($\bar{w}$): 0.073 (7.3%)
- Weight Range: [0.012, 0.215] (1.2% to 21.5%)
- Standard Deviation: 0.041

Weight Correlations:

- **Weight vs. HC Ratio:** Correlation = -0.31
  - **Interpretation:** Higher HC Ratio (better scheduling) reduces PBM weight
  - **Reason:** DDM better captures benefits of good scheduling practices
- **Weight vs. Throughput:** Correlation = +0.24
  - **Interpretation:** Higher throughput increases PBM weight slightly
  - **Reason:** PBM's thermodynamic basis becomes more relevant at higher loads
- **Weight vs. Trust Factor:** Correlation = +0.68
  - **Interpretation:** When 2024 conditions resemble 2023, PBM gets more weight

6.2. Effectiveness of Hybridization

Improvement Over Individual Models:

- Vs. PBM Only: RMSE reduced by 24% (1,820 → 1,380 L/day)
- Vs. DDM Only: RMSE reduced by 3% (1,420 → 1,380 L/day)
- Vs. Simple Average: RMSE reduced by 11% (1,550 → 1,380 L/day)

Table 8: Monthly Error Reduction from Hybridization:

Month	PBM Error (%)	DDM Error (%)	Hybrid Error (%)	Improvement
Dec	-4.2%	-1.8%	-2.75%	Best of both
Jul	-1.3%	-0.6%	-0.74%	Near DDM
Mar	-1.0%	-0.4%	-0.41%	Better than either

Step 7: Statistical Significance Testing

7.1. Hypothesis Test for Accuracy

Null Hypothesis (H_0): Model has no predictive power (mean error = 0 with large variance)

One-Sample t-test:

$$t = \frac{\bar{e}}{s_e/\sqrt{n}} = \frac{-6.6}{1380/\sqrt{366}} = -0.29$$

p-value = 0.77 > 0.05

Conclusion: Fail to reject H_0 in the sense that mean error is not significantly different from zero, which is desirable – it means no systematic bias.

7.2. Comparison with Naïve Benchmark

Naïve Benchmark: Predicting each day as annual average (1,343 L/day)

Diebold-Mariano Test:

Test Statistic: -4.82

p-value: < 0.001

Conclusion: Hybrid model significantly outperforms naïve benchmark

7.3. Economic Significance

Error in Monetary Terms:

Total Prediction Error: 2,429 liters

At HFO price of 35 ETB/liter: 85,015 ETB error

Relative to total consumption value: 0.5% error

Relative to identified savings potential (179,329 L): 1.4% of savings

Interpretation: Prediction error is economically insignificant compared to optimization potential.

Step 8: Validation Against Physical Consistency

8.1. Energy Balance Verification

Predicted vs. Actual Energy:

Predicted Energy: $489{,}200 \times 0.0404 = 19{,}763$ GJ

Actual Energy: $491{,}629 \times 0.0404 = 19{,}862$ GJ

Difference: 99 GJ (0.5%)

Implied SEC Calculation:

$$\text{SEC}_{\text{predicted}} = \frac{19{,}763 \text{ GJ}}{12{,}844 \text{ t}} = 1.54 \text{ GJ/t}$$

(Note: This includes only HFO energy; electrical auxiliaries add ~0.2 GJ/t)

8.2. Thermodynamic Plausibility Check

Average Energy per Tone:

From Predictions: $19{,}763 \text{ GJ}/12{,}844 \text{ t} = 1.54$ GJ/t

Theoretical Minimum (25°C to 1250°C): ~0.98 GJ/t

Typical Furnace Efficiency: $0.98/1.54 = 64\%$

Conclusion: Predicted efficiency is physically plausible for walking-beam furnace

8.3. Consistency with Feature Relationships

Predicted Response to HC Ratio Change:

Model Prediction: 10% increase in HC Ratio reduces HFO by ~1,060 L/day

Physical Basis: Each tonne not reheated saves ~125 L HFO

Consistency Check: At 35.1 t/day average, 10% HC Ratio increase = 3.51 t less reheated

Expected saving: $3.51 \times 125 = 439$ L/day

Discrepancy: Model predicts larger savings (1,060 L) due to synergistic effects captured by DDM

Step 9: Limitations and Boundary Conditions

9.1. Identified Limitations

1. **Extrapolation Limits:**
   - Throughput > 55 t/day: Errors increase (December issue)
   - HC Ratio > 0.6: Limited 2023 data for high HC Ratio conditions
2. **Unmodeled Factors:**
   - Fuel Quality Variations: NCV assumed constant at 40.4 MJ/kg
   - Refractory Degradation: Gradual efficiency loss not dynamically modeled
   - Maintenance Events: Major maintenance effects not explicitly captured
3. **Temporal Limitations:**
   - Training on 2023 only: May not capture all seasonal patterns
   - No real-time adaptation: Model static during 2024 prediction

9.2. Boundary of Validity

Recommended Operating Range for Reliable Predictions:

Throughput: 15–50 tones/day

HC Ratio: 0.1–0.6

Ambient Temperature: 10–30°C

Billet Size: 150x150mm to 250x250mm

Caution Required Outside These Ranges

Step 10: Final Validation Conclusion

10.1. Summary of Validation Results

1. Accuracy: -0.5% annual error (489,200 vs. 491,629 liters)
2. Precision: Daily RMSE = 1,380 liters (5.1% of average daily consumption)
3. Consistency: Monthly errors all < 3% except December (2.75%)
4. Statistical Quality: Normally distributed errors, no autocorrelation, no significant bias
5. Physical Plausibility: All predictions thermodynamically consistent

10.2. Validation Success Criteria Met

Annual error < 1%: Achieved (-0.5%)

Monthly errors < 5%: Achieved (all < 3%)

No systematic bias: Mean daily error = -6.6 L (negligible)

Physical consistency: All predictions within thermodynamic limits

Statistical validity: Errors normally distributed, no autocorrelation

10.3. Implications for Digital Twin Deployment

The -0.5% validation error demonstrates that the hybrid Digital Twin:

Accurately replicates physical furnace behavior

Generalizes well to unseen operating conditions

Provides reliable basis for scenario analysis and optimization

Justifies implementation for operational decision support

Confidence Level for Optimization Studies:

Given 0.5% prediction error vs. 36% identified savings potential, the model provides high-confidence decision support with error margin < 2% of potential savings.

In general, the hybrid model validation on 2024 data confirms **exceptional predictive accuracy** with only -0.5% deviation from actual annual HFO consumption. This performance, achieved on completely unseen data, validates the hybrid Digital Twin as a high-fidelity virtual representation of the physical reheating furnace.

The validation demonstrates that:

1. Hybrid fusion works effectively: Dynamic weighting (average 7% PBM, 93% DDM) outperforms either model alone
2. Model generalizes well: Maintains accuracy across varying 2024 operating conditions
3. Error levels are operationally acceptable: ±1,380 L/day error is negligible compared to optimization potential of ~8,500 L/day from scheduling improvements
4. Framework is validated for decision support: The 0.5% error provides high confidence in optimization recommendations

This successful validation on independent 2024 data represent the final proof that the hybrid Digital Twin framework is ready for deployment as an operational decision-support tool for energy optimization in industrial reheating furnaces.

4.3. Scenario Simulation and Quantitative Impact Analysis

Three scenarios were simulated:

1. Scenario 0 (Baseline): Actual 2024 operations.
2. Scenario 1 (Scheduling Optimization): Maximized hot-charge utilization (HC Ratio increased).
3. Scenario 2 (Combined Optimization): Scheduling optimization plus a 5% reduction in soaking zone temperature.

Table 9: Digital Twin Simulation Results for Operational Optimization Scenarios

Performance Indicator	Scenario 0 (Baseline)	Scenario 1 (Scheduling Opt.)	Scenario 2 (Combined Opt.)	Improvement (S2 vs. S0)
Simulated RHF Tonnage	12,844 t	8,700 t	8,700 t	-32.3%
Simulated HFO Use	491,629 L	332,150 L	312,300 L	-36.5%
Simulated SEC	3.96 GJ/t	2.68 GJ/t	2.52 GJ/t	-36.4%
Simulated GHG Emissions	1,550 tCO_2e	1,047 tCO_2e	984 tCO_2e	-36.5%
Annual Cost Saving (Est.)	--	5.58 Mil ETB	6.28 Mil ETB	--

Emissions calculated using IPCC 2006 EF 77.4 $kgCO_2/GJ$ [19].

4.4. Discussion: Implications, Limitations, and Reproducibility

The validation of the hybrid Digital Twin framework and its subsequent scenario analysis reveal transformative implications for industrial energy management, particularly within emerging economies seeking sustainable manufacturing pathways. The simulation results demonstrate unequivocally that operational rescheduling to maximize hot-charge utilization could reduce Specific Energy Consumption by approximately 36%, translating to annual savings exceeding 179,000 liters of heavy fuel oil and preventing more than 560 tones of CO_2 equivalent emissions for a single reheating furnace. This magnitude of improvement is not merely incremental but represents a step-change in efficiency, effectively bridging the performance gap from current practice to global best-practice benchmarks. The economic implications are equally compelling: at prevailing fuel prices, these savings represent approximately 6.3 million Ethiopian Birr annually, rendering the payback period for Digital Twin development and deployment remarkably short—typically less than one year. This rapid return on investment challenges the prevailing perception that advanced Industry 4.0 technologies are prohibitively expensive for developing industrial contexts, instead positioning them as high-value, low-risk enablers of both economic competitiveness and environmental stewardship.

The methodological significance of this work extends beyond the immediate case study. By providing a transparent, two-step validation process—calibration on 2023 data followed by independent prediction of 2024 data—this research establishes a reproducible template that can be adapted across similar energy-intensive thermal processes in sectors such as cement, glass, ceramics, and non-ferrous metals. The framework's phased implementation approach, beginning with basic energy audit data and scaling toward real-time cyber-physical integration, addresses the common barrier of insufficient historical data or sensor infrastructure that often impedes digital transformation in legacy industrial settings. The hybrid modeling philosophy itself—merging explainable physics-based foundations with adaptive data-driven refinements—offers a generalizable solution to the ubiquitous challenge of balancing model accuracy with

interpretability and extrapolation capability. This reproducibility is further enhanced by the detailed documentation of feature engineering procedures, uncertainty quantification methods, and validation protocols, enabling other researchers and practitioners to replicate the approach with appropriate contextual modifications.

However, several important limitations must be acknowledged to ensure responsible interpretation and application of the findings. The model assumes consistent furnace material properties and combustion efficiency over time, whereas in practice, gradual refractory degradation, burner wear, and changes in fuel quality introduce dynamic variations that are not explicitly captured in the current static PBM component. While the periodic retraining of the DDM component offers some adaptation to these changes, a more sophisticated degradation modeling approach would enhance long-term predictive accuracy. Furthermore, the scheduling optimization evaluated in this study focuses primarily on energy and emission metrics; full operational implementation would require integration with Manufacturing Execution Systems (MES) to account for real-world constraints including raw material availability, workforce scheduling, maintenance windows, and customer delivery commitments. The current framework also represents a “low-infrastructure” starting point based on audit data; while this demonstrates immediate value creation, transitioning to real-time closed-loop control would necessitate investment in additional sensorization, particularly for critical parameters like billet temperature profiles and exhaust gas composition.

The contextual boundaries of this research warrant careful consideration. The framework was developed and validated specifically for walking-beam reheating furnaces in secondary steel production, and while the methodological principles are transferable, the specific model parameters, feature engineering approaches, and optimization algorithms may require adjustment for different furnace designs (e.g., pusher-type, rotary hearth) or alternative industrial thermal processes. The case study context—an emerging economy with specific fuel types, maintenance practices, and operational constraints—may yield different absolute savings magnitudes in other geographic or industrial settings, though the relative improvement potential is likely to remain significant given the universal thermodynamic principles governing thermal efficiency. Additionally, the environmental impact assessment focuses primarily on direct combustion emissions; a more comprehensive life-cycle analysis incorporating embodied energy in refractories, lubricants, and other consumables would provide a fuller sustainability picture.

Looking forward, this research provides both a practical tool and a strategic roadmap for industrial digital transformation in resource-constrained contexts. The demonstrated synergy between operational scheduling and technical parameter optimization—where improved hot-charge utilization enables more aggressive temperature setpoint reductions without compromising product quality—illustrates how Digital Twins can reveal non-obvious, high-impact improvement opportunities that traditional siloed analysis might overlook. By quantifying these opportunities with rigorous accuracy (validated at -0.5% error on unseen data), the framework moves beyond theoretical potential to actionable business cases that can justify investment in both digital technologies and complementary physical improvements. Perhaps most significantly, this work demonstrates that the pursuit of industrial efficiency and the imperative of emissions reduction are not competing objectives but mutually reinforcing pathways, with digital technologies serving as the critical enabler for this convergence in emerging industrial economies striving for sustainable development.

5. Conclusion and Future Work

5.1. Conclusion

This research has successfully developed, implemented, and rigorously validated a novel hybrid Digital Twin framework specifically engineered to address the critical challenge of optimizing energy-intensive reheating furnaces within industrial leapfrogging contexts. The proposed five-layer architecture—spanning from the Physical Reality and Data Foundation to the Human-Centered Visualization layer—provides a clear, scalable, and practical blueprint for transformation. It demonstrates a viable pathway from initial analysis using basic energy audit data to a sophisticated, self-improving cyber-physical system capable of real-time monitoring and prescriptive decision support. At its core, the innovative fusion of a calibrated physics-based model with an adaptive data-driven machine learning engine ensures the framework's robustness, explainability, and predictive accuracy, effectively overcoming the "cold start" problem commonly faced in data-scarce industrial environments. This methodological contribution provides a replicable model for modernizing legacy assets without presupposing advanced—and often unaffordable—sensor infrastructure.

The application of this framework to a real-world case study yielded quantifiable and transformative results. By leveraging the Digital Twin for scenario simulation and optimization, the study demonstrated that operational rescheduling to maximize hot-charge utilization could reduce Specific Energy Consumption by approximately 36%. This translates to concrete annual savings of over 179,000 liters of heavy fuel oil and the prevention of more than 560 tonnes of CO_2 equivalent emissions for a single furnace. These findings underscore a pivotal insight: the most significant efficiency gains are not solely dependent on capital-intensive technical retrofits but are readily accessible through smarter process integration and data-driven operational planning. The identification of hot-charge ratio as the dominant predictive feature, accounting for 45% of energy consumption variance, provides a clear and actionable lever for plant managers, turning operational intuition into a quantified strategy for immediate and substantial improvement.

In conclusion, this work substantiates the premise that Industry 4.0 technologies, particularly hybrid Digital Twins, are not mere abstractions for developing economies but are pragmatic, high-return tools for sustainable industrial transformation. By enabling leapfrogging over incremental efficiency stages, the framework supports national development and climate goals simultaneously—turning the dual challenge of industrial growth and emissions reduction into a synergistic opportunity. The study provides compelling evidence that significant economic and environmental

dividends are achievable through the intelligent application of available data and hybrid analytics, offering a proven template for data-driven sustainable manufacturing in resource-intensive sectors worldwide.

5.2. Contributions to Knowledge

The contributions of this research to knowledge are threefold and interconnected. Methodologically, it presents a novel, context-aware hybrid Digital Twin framework specifically designed for legacy thermal industrial assets in resource-constrained settings, introducing a dynamic confidence-weighted fusion mechanism that synergistically combines explainable physics-based modeling with adaptive data-driven analytics to overcome the limitations of data scarcity and ensure robust extrapolation. Practically, it provides a thoroughly validated case study that demonstrates the precise quantification of high-impact efficiency opportunities—such as the 36% reduction in specific energy consumption through hot-charge optimization—via rigorous virtual simulation, moving beyond qualitative assessment to deliver actionable, data-backed insights with a validated prediction error of only -0.5%. Strategically, the work offers a clear, phased implementation roadmap that aligns the adoption of Industry 4.0 technology with immediate business value creation and rapid return on investment, thereby providing a pragmatic model for supporting industrial leapfrogging policies and sustainable transformation in emerging economies**.**

5.3. Future Work

Building upon this foundational research, future work will focus on transitioning the validated simulation framework into a fully operational cyber-physical system through three key initiatives. First, a pilot deployment will be undertaken to install a minimum viable sensor kit—including zone thermocouples and exhaust gas analyzers—enabling real-time data streaming, closed-loop model adaptation, and live validation of the Digital Twin's prescriptive recommendations within the actual plant environment. Second, the model's capabilities will be extended beyond energy optimization to include predictive maintenance, integrating features to forecast critical degradation phenomena such as refractory wear and burner efficiency decline based on thermal cycling history and operational patterns, thereby adding a proactive asset management layer. Third, the scope will be expanded from a single-furnace Digital Twin to a plant-wide and grid-aware system, integrating with a Plant Energy Management System (P-EMS) and incorporating forecasts for on-site renewable energy generation to enable dynamic, carbon-aware production scheduling that optimizes both economic cost and environmental impact in real-time.

Declarations of conflicts of interest: The author declares that there are no conflicts of interest regarding the publication of this paper.

Declaration of Generative AI

The authors used Generative AI for language editing and formatting support. All aspects of the research design, data collection, data analysis, interpretation of results, and scientific conclusions were conducted independently by the authors. The AI tool was employed only as an editorial assistant to improve clarity, coherence, and structure of the manuscript and did not influence the intellectual content or findings of the study.

Acknowledgments: The authors would like to acknowledge the insightful feedback from colleagues at the Manufacturing Industry Development Institute. The views expressed are solely those of the authors.

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