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Pseudo Projective φ-Recurrent Curvature Tensor and Speacial(ε, δ)-Trans Sasakian Manifold

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Abstract. In object of the present papers this paper is study pseudo projective ϕ- recurrent curvature tensor (ε, δ) –Trans Sasakian Manifold.

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Key words: (ε, δ)-Trans-Sasakian manifold, pseudo projective , φ-recurrent manifolds.

Introduction: The notion of Local symmetry of a Reimannian manifold has been studied by many auother in several ways to a different extent. As a weaker version of local symmetry.1977. Takahashi,[23] introduced the notion of locally φ-symmetric Sasakian manifold and obtained theire several intersiting results.The Properties of pseudo projective curvature tensor is studied by many geometry [17], [24], [25], [21] and obtained theire some interesting results. In this paper we shown that pseudo projective recurrent Sasakian manifold. The characteristic vector field ξ and the vector ρ associated to the1-form a are co-directional. Finally we prove that a three dimensional locally pseudo –projective ϕ-recurrent manifold is of constant curvature.

The notion of locally φ-symmetric Sasakian manifold and pseudo projective curvature tensor on a Riemannian manifold were studied by the author [2] and [9] respectively. Also J. A. Oubina in 1985 introduced a new class of almost contact metric structures which was a generalization of Sasakian [25], α-Sasakian [9] Kenmotsu [16], β-Kenmotsu and cosymplectic manifolds, which was called Trans-Sasakian manifold [23].After him many authors [23], [24], [25] have studied various type of properties in Trans-Sasakian manifold. In this paper we have studied pseudo projective φ -recurrent (ε, δ)-Trans Sasakian manifold which satisfying is an Einstein manifold. $\varphi(grad\alpha) = (2n-1)grad\beta$, and found that the manifold is an Einstein manifold. Further it is proved that in a pseudo projective φ-recurrent (ε, δ)- trans –Sasakian manifold (M^{2n+1}, g), $n \geq 1$, the characteristic vector field ξ, ρ associated to the 1-form are opposite directional.

Preliminaries. In this section some special notion, results and more background material are presented which we will use later .Let M be an (2n+1)-dimensional differentiable manifold with almost contact structure (ϕ, ξ, η, g), where $\phi$is a (1.1) tensor field, ξ is a vector field, and η is a 1-form on M satisfying

$$\phi^2(X) = -X + \eta(X)\xi, \ \eta(\xi) = 1, \qquad (1)$$

If there exists semi-Riemannian metric g satisfying

$$g(\phi X, \phi Y) = g(X,Y) - \varepsilon\eta(X)\eta(Y), \qquad (2)$$

for all vector fields X, and Y .where $\varepsilon = \pm 1$, then structures (ϕ, ξ, η, g) is called then an $\varepsilon -$ almost contact metric structures and M is called an (ε)-contact manifold .for an $\varepsilon -$ contact manifold we have

$$g(X,\phi Y) = -g(\phi X, Y), \ \ \varepsilon g(X, \xi) = \eta(X) \text{ or } \varepsilon = g(\xi, \xi) \qquad (3)$$

If $\varepsilon = -1$, and index of g is odd then M is a space like Sasakian manifold. Further M is The usual Sasakian manifold for $\varepsilon = 1$, and index of g as 0 and M is a Lorentzian Sasakian manifold. In J.A. Oubina introduced the notion of a trans-Sasakian manifold. An the almost contact metric manifold M is trans –Sasakian manifold. If then exists two function α and β on M such that

$$(\nabla_X \phi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X), \qquad (4)$$

For all vector fields X and Y on M. Throughout this paper we study a class of almost contact manifolds called (ε, δ)-trans-Sasakian manifolds. An almost contact metric manifold M is a (ε, δ)-trans-Sasakian manifold.

$$(\nabla_X \phi)Y = \alpha(g(X, Y)\xi - \varepsilon\eta(Y)X) + \beta(g(\phi X, Y)\xi - \delta\eta(Y)\phi X), \qquad (5)$$

For all vector field X and Y on M, where α andβ are some function on M and $\varepsilon = \pm 1, \delta = \pm 1$. If β=0 and α=1, (β=1 andα=0), then manifold reduces to (ε)-Sasakian (or δ-Kenmotsu manifolds).

From equation (11), we have

$$\nabla_X \xi = \varepsilon\alpha\phi X - \beta\delta\phi^2 X\,, \qquad (6)$$

$$(\nabla_X \eta)Y = \alpha g(\phi X, Y) + \varepsilon\delta g(\phi X, Y) + \varepsilon\beta\delta g(\phi X, \phi Y)\,, \qquad (7)$$

where ∇ denote the Riemannian connection of g, in an n-dimensional kenmotsu manifold, we have

$$R(X, Y)\xi = (\alpha^2 - \beta^2)(\eta(Y)X - \eta(X)Y + 2\alpha\beta(\eta(Y)\phi X - \eta(X)\phi Y$$

$$+\varepsilon((Y\alpha)\phi X - (X\alpha)\phi Y) + \delta((Y\beta)\phi^2 X - (X\beta)\phi^2 Y), \qquad (8)$$

$$R(X, Y)\xi = \varepsilon[(\alpha^2 - \beta^2) + \left(\frac{2-\varepsilon}{2}\right) r][\eta(Y)X - \eta(X)Y], \qquad (9)$$

$$S(X, \xi) = (2n(\alpha^2 - \beta^2) - \xi\beta)\eta(X) - (2n - 1)\text{grad}\beta + \phi(\text{grad}\alpha), \qquad (10)$$

$$Q\xi = (2n(\alpha^2 - \beta^2) - \xi\beta)\xi - (2n - 1)\text{grad}\beta + \phi(\text{grad}\alpha)\,, \qquad (11)$$

$$\xi\alpha + 2\alpha\beta = 0, \qquad (12)$$

when $\phi(grad\alpha) = (2n-1)grad\beta$ then equations (10) and (11), we have

$$S(X,\xi) = 2n(\alpha^2 - \beta^2)\eta(X) \tag{13}$$

$$R(X,Y)\xi = \left(2A - \frac{r}{2}\right)g(Y,Z)\eta(X) - g(X,Z)\eta(Y) + B(g(Y,Z)\eta(X) - g(X,Z)\eta(Y)) \tag{14}$$

The sectional curvature of the plan section spanned by vector X and Y is given by

$$K(X,Y) = \frac{R(X, Y, X, Y)}{g(X, X)g(Y, Y) - g(X, Y)^2} \tag{15}$$

Totally real bisectional curvature of totally real section {XY}, where X and Y are orthonormal to ξ and $\phi\{X,Y\} \perp \{X,Y\}$ is given by

$$B(X,Y) = R(X, \phi X, Y, \phi Y) \tag{16}$$

If X is orthonormal to ξ then the plane section $\{X, \phi X\}$ is called ϕ-section and curvature associated with this section is called ϕ-sectional curvature which then given by

$$H(X) = K(X,\phi X) = R(X,\phi X, X, \phi X) \tag{17}$$

$$Q\xi = 2n(\alpha^2 - \beta^2)\xi. \tag{18}$$

Again a (ε,δ) –trans-Sasakian manifold is be locally ϕ -symmetric if

$$\phi^2((\nabla_W R)(X,Y)Z) = 0, \tag{19}$$

For all vector fields X, Y, Z, W orthogonal to ξ. A (ε,δ)-trans-Sasakian manifold is to be pseudo projective ϕ –recurrent manifold if there exist A non-zero 1-form A such that

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)\overline{P}(X,Y)Z, \tag{20}$$

For X, Y, Z, WϵTM, where $\overline{P}$ is a pseudo projective curvature tensor given by[],

$$\overline{P}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{2n+1}\left[\frac{a}{2n} + b\right][g(Y,Z)X - g(X,Z)Y]. \tag{21}$$

Where a,b are constants such that a,b≠ 0, R is the curvature tensor S is the Ricci-tensor and r is the scalar curvature. Also,

$$g(QX,Y) = S(X,Y), \tag{22}$$

Q being the symmetric endomorphism of the tangent space at each point corresponding to the Ricci-tensor S. The 1-form A is defined as

$$g(X,\rho) = A(X), \forall X \epsilon TM. \tag{23}$$

being the vector field associated to the1-form A. The above results will be useful in the next section.

3. Pseudo Projective φ-recurrent (ε,δ) -trans Sasakian Manifold.

In this section we consider a (ε, δ)-trans Sasakian manifold which is pseudo-projective φ-recurrent. Then by virtue of (20) and (1) we have

$$(\nabla_W \bar{P})(X, Y)Z + \eta((\nabla_W \bar{P})(X, Y)Z)\xi = A(W)\bar{P}(X, Y)Z \,. \tag{24}$$

From (24) it follows that

$$g(\nabla_W \bar{P})(X, Y)Z, U) + \eta((\nabla_W \bar{P})(X, Y)Z)\eta(U) = A(W)g(\bar{P}(X, Y)Z, U)\,. \tag{25}$$

Let $\{e_i\}, i = 1,2, \dots \dots \dots, 2n+1$, be an orthonormal basis of tangent space at any point be an of the manifold. then putting X=U=e_i in (25) and taking summation over i,1≤ i ≤2n+1,we get

$$(\nabla_W S)(Y, Z) = A(W)[S(Y, Z) - \frac{r}{2n+1} g(Y, Z)]\,. \tag{26}$$

Replacing Z=ξ and using equations (13) and (3) we have

$$(\nabla_W S)(Y, \xi) = A(W)[2n(\alpha^2 - \beta^2) - \frac{r}{\varepsilon(2n+1)} \eta(Y)]\,. \tag{27}$$

Now we Know $(\nabla_W S)(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi)$. Using equations (12)

And (6) in the above relation we get, after a brief calculation

$$(\nabla_W S)(Y, \xi) = 2n(\alpha^2 - \beta^2)[-\alpha g(X, \phi W) + \beta\delta\varepsilon g(\phi X, \phi W)] - 4\beta\delta n(\alpha^2 - \beta^2)\eta(W)$$

$$+ 2\varepsilon\alpha n(\alpha^2 - \beta^2)\eta(\phi W) + \varepsilon\alpha S(Y, \phi W) - \beta\delta S(Y, W) + 2\beta\delta n(\alpha^2 - \beta^2)\eta(Y)\eta(W)\,. \tag{28}$$

By virtue of (2) and (28) reduces to

$$(\nabla_W S)(Y, \xi) = 2n(\alpha^2 - \beta^2)[-\alpha g(Y, \phi W) + \beta\delta\varepsilon\, g(Y, W)] - 4\beta\delta n(\alpha^2 - \beta^2)\eta(W)$$

$$- \beta\varepsilon^2\delta\eta(Y)\eta(W) + 2\varepsilon\alpha n(\alpha^2 - \beta^2)\eta(\phi W) + \varepsilon\alpha S(Y, \phi W) - \beta\delta\, S(Y, W)$$

$$+ 2\beta\delta n(\alpha^2 - \beta^2)\eta(Y)\eta(W)\,. \tag{29}$$

Now, from the using equations (27) and (29),we get

$$2n(\alpha^2 - \beta^2)[-\alpha g(Y, \phi W) + \beta\delta\varepsilon g(Y, W)] - 4\beta\delta n(\alpha^2 - \beta^2)\eta(W)$$

$$- \beta\varepsilon^2\delta\eta(Y)\eta(W) + 2\varepsilon\alpha n(\alpha^2 - \beta^2)\eta(\phi W) + \varepsilon\alpha S(Y, \phi W) - \beta\delta S(Y, W)$$

$$+ 2\beta\delta n(\alpha^2 - \beta^2)\eta(Y)\eta(W) = A(W)[2n(\alpha^2 - \beta^2) - \frac{r}{\varepsilon(2n+1)} \eta(Y)]\,. \tag{30}$$

Replacing Y and W by Y and ϕY and ϕW Respectively in (30) and then using (1), (3), (13) and (22), we obtain

$$2n(\alpha^2 - \beta^2)[-A(\phi W) + \alpha g(\phi Y, \phi^2 W) + \beta\delta\varepsilon g(\phi Y, \phi W)] - 4\beta\delta n(\alpha^2 - \beta^2)\eta(\phi W)$$

$$- \beta\varepsilon^2\delta\eta(\phi Y)\eta(\phi W) + 2\varepsilon\alpha n(\alpha^2 - \beta^2)\eta(\phi^2 W) + \alpha\varepsilon S(\phi Y, \phi^2 W) - \beta\delta\, S(\phi Y, \phi W)$$

$$+ 2\beta\delta n(\alpha^2 - \beta^2)\eta(\phi Y)\eta(\phi W) = A(W)[-\frac{r}{\varepsilon(2n+1)} \eta(\phi Y)]\,.$$

$$2n(\alpha^2-\beta^2)[\alpha g(\phi Y,W)+\beta\delta\varepsilon g(\phi Y,\phi W)]\ +\alpha\varepsilon S(\phi Y,W)-\beta\delta\, S(\phi Y,\phi W)\ =0$$

$$\Rightarrow\qquad 2n(\alpha^2-\beta^2)g(\phi Y,W)=-\varepsilon S(\phi Y,W) \tag{31}$$

And

$$\Rightarrow\qquad 2n(\alpha^2-\beta^2)\varepsilon g(\phi Y,\phi W)=\ S(\phi Y,\phi W) \tag{32}$$

Thus we can states as follows.

Theorem (3.1). A pseudo projective ϕ- recurrent (ε,δ) trans-sasakian manifold. (M^{2n+1},g)

satisfying $\phi(grad\alpha)$=(2n-1)gradβ, is an Einstein manifold.

Now from the above equations (34) and (21), we have

$$(\nabla_W\bar{P})(X,Y)Z=\eta\big((\nabla_W\bar{P})(X,Y)Z\big)\xi-A(W)\bar{P}(X,Y)Z. \tag{33}$$

$$a(\nabla_W R)(X,Y)Z+b[\nabla_W S(Y,\ Z)X-\nabla_W S(X,Z)Y]-\frac{r}{2n+1}\left[\frac{a}{2n}+b\right][\nabla_W g(Y,Z)X-\nabla_W g(X,Z)Y]$$

$$=a(\nabla_W R(X,Y)Z)+b[\nabla_W S(Y,Z)\eta(X)-\nabla_W S(X,Z)\eta(Y)]$$

$$-\frac{r}{2n+1}\left[\frac{a}{2n}+b\right][\nabla_W g(Y,Z)\eta(X)-\nabla_W g(X,Z)\eta(Y)]\xi$$

$$-A(W)\bar{P}\,(X,Y)Z$$

$$a(\nabla_W R)(X,Y)Z=\ a(\nabla_W R(X,Y)Z)+b[\nabla_W S(Y,Z)\eta(X)\xi-\nabla_W S(X,Z)\eta(Y)\xi]$$

$$-b[\nabla_W S(Y,Z)X-\ \nabla_W S(X,Z)Y]+\frac{r}{2n+1}\left[\frac{a}{2n}+b\right][\nabla_W g(Y,Z)X-\nabla_W g(X,Z)Y]$$

$$-\frac{r}{2n+1}\left[\frac{a}{2n}+b\right]\begin{bmatrix}\nabla_W g(Y,Z)\eta(X)\xi-\nabla_W g(X,Z)\eta(Y)\xi\\ -A(W)\{g(Y,Z)X-g(X,Z)Y\end{bmatrix}$$

$$-A(W)\ \{a(R(X,Y)Z)+b[S(Y,Z)X-S(X,Z)Y]\}$$

Taking inner product with ξ, we get

$$a\ A(W)\ \eta(R(X,Y)Z)=b[\nabla_W S(Y,Z)\eta(X)-\nabla_W S(X,Z)\eta(Y)]$$

$$-b\{\nabla_W S(Y,Z)\eta(X)-\ \nabla_W S(X,Z)\eta(Y)+A(W)\ [S(Y,Z)\eta(X)-S(X,Z)\eta(Y)]\}$$

$$+\frac{r}{2n+1}\left[\frac{a}{2n}+b\right][\nabla_W g(Y,Z)\eta(X)-\nabla_W g(X,Z)\eta(Y)-\ \nabla_W g(Y,Z)\eta(X)$$

$$+\nabla_W g(X,Z)\eta(Y)-A(W)\{g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}]. \tag{33}$$

Now from equation (33) by cyclic permutations X,Y,Z, we get

$$a\ A(X)\ \eta(R(Y,W)Z)=b[\nabla_X S(W,Z)\eta(Y)-\nabla_X S(Y,Z)\eta(W)]$$

$$-b\{\nabla_X S(W,Z)\eta(Y)-\ \nabla_X S(Y,Z)\eta(W)+A(W)\ [S(W,Z)\eta(Y)-S(Y,Z)\eta(W)]\}$$

$$+\frac{r}{2n+1}\left[\frac{a}{2n}+b\right][\nabla_X g(W,Z)\eta(Y)-\nabla_X g(Y,Z)\eta(W)-\nabla_X g(W,Z)\eta(Y)$$

$$+\nabla_X g(Y,Z)\eta(W)-A(W)\{g(W,Z)\eta(Y)-g(Y,Z)\eta(W)\}] \qquad (34)$$

$$a\,A(Y)\,\eta(R(W,X)Z)=b[\nabla_Y S(X,Z)\eta(W)-\nabla_Y S(W,Z)\eta(X)]$$

$$-b\{\nabla_Y S(X,Z)\eta(W)-\nabla_Y S(X,Z)\eta(W)+A(W)\,[S(X,Z)\eta(W)-S(W,Z)\eta(X)]\}$$

$$+\frac{r}{2n+1}\left[\frac{a}{2n}+b\right][\nabla_Y g(X,Z)\eta(W)-\nabla_Y g(W,Z)\eta(X)-\nabla_Y g(X,Z)\eta(W)$$

$$+\nabla_Y g(W,Z)\eta(X)-A(W)\{g(X,Z)\eta(W)-g(W,Z)\eta(X)\}] \qquad (35)$$

Using the Bianchi's identity in equations (33),(34) and (35),we get

$$a\,A(W)\,\eta(R(X,Y)Z)+aA(X)\,\eta(R(Y,W)Z+\ a\,A(Y)\,\eta(R(W,X)Z$$

$$=A(W)[\{S(W,Z)X-S(X,Z)W\}-\frac{r}{2n+1}\left[\frac{a}{2n}+b\right]c\{g(X,Z)\eta(W)-g(W,Z)\eta(X)\}] \qquad (36)$$

Putting Y=Z=e_i ,where be orthonormal basis of the tangent space at any point of the manifold, in (36) and taking summation over I,1≤ i ≤2n+1, we get

$$A(W)+aA(X)=A(W)\,(1\text{-}a)S(X,W)\text{-}a(2n+1)S(W,X) \qquad (37)$$

Putting again X=ξ and using equations (1) and (3), we get

$$A(W)=\frac{-a(2n+1)}{(1+2a)}-\frac{aA(\xi)}{\varepsilon(1+2a)S(W,\xi)} \qquad (38)$$

For any vector W and being the vector field associated to the 1-form A, defined as (18). From equation (38), we can states as follows**.**

Theorem(3.2): In pseudo projective ϕ-recurrent(ε,δ) trans-Sasakian manifold $(M^{2n+1},g),n\geq 1$ the characteristic vector field ξ and the vector field ρ to the A are opposite directional and the 1-form A is given.

Example: Consider 3-dimensional (ε,δ)-trans-sasakian manifold M={(x,y,z)ϵR^3;Z≠ 0}, where

(x,y,z) are the standard coordinates in R^3. Let $\{e_1,e_2,e_3\}$ be linearly independent given by

$$e_1=z\frac{\partial}{\partial x},e_2=z\frac{\partial}{\partial y},e_3=-z\frac{\partial}{\partial z}. \qquad (75)$$

Let g be the Riemannian metric defined by $g(e_1,e_2)$=$g(e_2,e_3)$=$g(e_1,e_3)$=1,

$$g=\frac{1}{z^2}(dx\otimes dx+dy\otimes dy+dz\otimes dz). \qquad (76)$$

The (ϕ,ξ,η) structure is given by

$$\eta=-\frac{1}{z}dz,\ \xi=e_3=-z\frac{d}{\partial z}, \qquad (77)$$

$$\phi e_1=-e_2,\quad \phi e_2=e_1,\quad \phi e_3=0.$$

The linearity property of ϕ and g yields that

$$\eta(E_3) = 1, \phi^2 U = -U + \eta(U)e_3, g(\phi U, \phi W)=g(U,W)-\eta(U)\eta(W),$$

for any vector fields U, W on M. by definition of Lie bracket, we have

$$[e_1, e_2]=0, \quad [e_1, e_2]=e_1, \quad [e_2, e_3] = e_2. \tag{78}$$

Let ∇ be the Levi-civita connection with respect to above metric g is given by Koszula formula

$$2g(\nabla_X Y, Z)=X(g(Y,Z))+Y(g(Z,X))-Z(g(X,Y))-g(X,[Y,Z])-g(Y,[X,Z])+g(Z,[X,Y]), \tag{79}$$

And by virtue of it we have

$$\nabla_{e_1} e_3 = e_1, \qquad \nabla_{e_2} e_3 = e_2, \qquad \nabla_{e_3} e_3 = 0,$$

$$\nabla_{e_1} e_3 = e_1, \qquad \nabla_{e_2} e_2 = -e_3, \qquad \nabla_{e_3} e_3 = 0, \tag{80}$$

$$\nabla_{e_1} e_1 = -e_3, \quad \nabla_{e_1} e_1 = 0, \qquad \nabla_{e_3} e_1 = 0,$$

Clearly equation (8) shows that (ϕ, ξ, η, g) satisfies (10), (11) and (12). Thus M is a (ε, δ) $-$trans Sasakian manifold. It is known that

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z\,. \tag{81}$$

With the help of equations (80) and (81), it can be easily verified that

$$R(e_1, e_2)e_2 = -e_1, \qquad R(e_1, e_3)e_3 = -e_1,$$

$$R(e_1, e_1)e_1 = 0, \qquad R(e_2, e_1)e_1 = -e_2,$$

$$R(e_2, e_3)e_2 = -e_1, \qquad R(e_2, e_2)e_2 = 0,$$

$$R(e_1, e_1)e_1 = -e_3, \qquad R(e_3, e_2)e_2 = -e_3,$$

$$R(e_3, e_3)e = 0. \tag{82}$$

From the above expression of the curvature tensor we obtain

$$S(e_1, e_1)= g(R(e_1, e_2)e_2, \; e_1) + g(R(e_1, e_3)\, e_3, e_1)= -2. \tag{83}$$

Similarly we have

$$S(e_2, e_2)= S(e_3, e_3) = -2$$

$$\mathcal{L}_\xi g(e_i, e_i)=2[g(e_i, e_i) - \eta(e_i)\eta(e_i)]. \tag{84}$$

Now by X=Y=e_i in (1), where i=1,2, 3 and by virtue of above equation we get. The value of λ which is strictly greater than 0. Thus this is an example of expanding Ricci solitons in (ε, δ) $-$trans Sasakian manifolds.

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