IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

A NOVEL FORM OF HEPTAGONAL NEUTROSOPHIC b- OPEN SETS

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Abstract : The aim of this paper is to discover and present the notion of heptagonal neutrosophic b-open sets in heptagonal neutrosophic topological spaces. We have also putforth a few of its characteristics by introducing and discussing the idea of heptagonal neutrosophic b-interior and heptagonal neutrosophic b- closure operators. We also define b-open sets using neutrosophic membership functions and their properties such as union, intersection and complementation.

IndexTerms: Heptagonal Neutrosophic Topology, Heptagonal Neutrosophic b-open Set, Heptagonal Neutrosophic b-Interior and Heptagonal Neutrosophic b-Closure

INTRODUCTION

Neutrosophic topological spaces have applications in various fields such as decision-making, computer science, and engineering, where the presence of indeterminate, vague, or uncertain information is prevalent. They provide a powerful tool for modeling and analyzing complex systems where classical topological spaces may not be sufficient. Subsequently after Zadeh's introduction of the fuzzy set in the year 1965 with the membership function, the aforesaid fields are developed in various phases with many real life situations. The investigator focused their research in the above fields towards applications in practical problems with the help of intuitionistic fuzzy numbers with membership and non-membership values which was developed by Atanassov.K.T in 1986. There was a new finding between membership and non-membership values called indeterminacy and combined three values named as neutrosophic numbers which was introduced by Smarandache in 2005.

NEED OF THE STUDY.

While neutrosophic sets have been widely studied and applied in various fields, the existing literature primarily focuses on traditional neutrosophic models, such as triangular and cubic neutrosophic sets. However, there is a noticeable lack of in-depth exploration into higher-dimensional neutrosophic structures, particularly the heptagonal neutrosophic sets. The introduction of a heptagonal structure offers the potential for richer representations of uncertainty, indeterminacy, and vagueness, which are critical in applications requiring more nuanced decision-making models. This study aims to fill the gap by investigating the properties, operations, and applications of heptagonal neutrosophic sets, contributing to the development of more advanced and flexible tools for handling complex, real-world problems.

RESEARCH METHODOLOGY

The methodology employed in this study involves a combination of theoretical analysis, construction of new models, and exploration of mathematical properties of heptagonal neutrosophic b-open sets within the context of topology.

2. Preliminaries

2.1 Heptagonal neutrosophic topology:

A heptagonal neutrosophic number S is defined and described as S =< [(p, q, r, s, t, u, v); μ] , [(p′ , q′ , r′ , s′ , t′ , u′ , v′); γ] , [(p″, q″, r″, s″, t″, u″, v″); η] > where $\mu, \gamma, \eta \in [0, 1]$.The truth membership function $\rho : R \to [0, \mu]$, the indeterminacy membership function $\sigma : R \to [\gamma, 1]$, the falsity membership function $\omega : R \to [\eta, 1]$. Using ranking technique of heptagonal neutrosophic number is changed as,

$$\rho(x) = \frac{(p+q+r+s+t+u+v)}{7}$$

$$\sigma(x) = \frac{(p' + q' + r' + s' + t' + u' + v')}{7}$$

$$\omega(x) = \frac{(p'' + q'' + r'' + s'' + t'' + u'' + v'')}{7}$$

2.2 Definition for Neutrosophic topology

Let $\tau \subseteq N(X)$, then τ is called a neutrosophic topology on X if

i. X and $\emptyset$ belong to τ ,

ii. The union of any number of neutrosophic sets in τ belongs to τ ,

iii. The intersection of any two neutrosophic sets in τ belongs to τ . The pair (X, τ) is called a neutrosophic topological space over X.

Moreover, the members of τ are said to be neutrosophic open sets in X. If $A^c \in \tau$ then $A \in N(X)$ is said to be neutrosophic closed set in X.

2.3 Definition :

A subset A of a space X is said to be:

1. Semi-open if $A \subseteq Cl(Int(A))$
2. Pre open if $A \subseteq Int(Cl(A))$
3. b-open if $A<Int(Cl(A)) \cup Cl(Int(A))$

2.4 Definition for b-open set:

The closure and the interior of A of X are denoted by Cl(A) and Int(A),respectively. A subset A of X is said to be b-open if **$A<Int(Cl(A)) \cup Cl(Int(A))$** **3 .HN b-open sets**

Definition 3.1: Let A_{HN} be a HNS of a HNTS X. Then A_{HN} is said to be a Heptagonal Neutrosophic b-open [written HN-BO] set of X if there exists a heptagonal neutrosophic open set HNO such that $A_{HN} \subseteq HNCl(HNO) \cup HNInt(HNO)$.

Theorem 3.2: A subset A_{HN} in a HNTS X is a HN-b open set iff $A_{HN} \subseteq HNCl(HNInt(A_{HN})) \cup HNInt(HNCl(A_{HN}))$

Proof:

Necessity: Let A_{HN} be a HN-b-open set in X. Then $A_{HN} \subseteq HNCl(HNO) \cup HNInt(HNO)$for some heptagonal neutrosophic open set HNO But $HNO \subseteq HNInt(A_{HN}) \cup HNCl(A_{HN})$ thus $HNCl(HNO) \subseteq HNCl(HNInt(A_{HN})) \cup HNInt(HNCl(A_{HN}))$.

Hence $A_{HN} \subseteq HNCl(HNO) \cup HNInt(HNO)$.

$A_{HN} \subseteq HNCl(HNInt(A_{HN})) \cup HNInt(HNCl(A_{HN}))$.

Sufficiency: Let $A_{HN} \subseteq HNCl(HNInt(A_{HN})) \cup HNInt(HNCl(A_{HN}))$.

Since $HNO = HNInt(A_{HN}) \cup HNCl(A_{HN})$. we have $HNO \subseteq A_{HN} \subseteq HNCl(HNO) \cup HNInt(HNO)$ Hence A_{HN} is a HN-b-open set.

Theorem 3.3 : Let (X, τ) be a HNTS. Then union of two HN-b-open sets is again a HN- b-open set in the HNTS X.

Proof: Let A_{HN} and B_{HN} are HN-b open sets in X.

Then $A_{HN} \subseteq HNCl(HNInt(A_{HN})) \cup HNInt(HNCl(A_{HN}))$ and $B_{HN} \subseteq HNCl(HNInt(B_{HN})) \cup HNInt(HNCl(B_{HN}))$

Therefore $A_{HN} \cup B_{HN} \subseteq [HNCl(HNInt(A_{HN})) \cup HNInt(HNCl(A_{HN}))] \cup [HNCl(HNInt(B_{HN})) \cup HNInt(HNCl(B_{HN}))]$

$A_{HN} \cup B_{HN} = HNCl[(HNInt(A_{HN}) \cup HNInt(B_{HN})) \cup HNInt(HNCl(A_{HN} \cup HNCl(B_{HN})]$

$\subseteq HNCl[HNInt(A_{HN} \cup B_{HN})] \cup HNInt[HNCl(A_{HN} \cup B_{HN})]$

[By Theorem 3.2],

Hence $A_{HN} \cup B_{HN}$ is a HN-b- open set in X.

Theorem 3.4: Let (X, τ) be a HNTS. Then union of a finite collection of HN-b-open sets is again a HN- b- open set in the HNTS X.

Proof: For each $i\in\triangle$, $(A_{HN})i$is a HN-b- open sets in X. Then by theorem 3.2, $(A_{HN})i\subseteq HNCl(HNInt((A_{HN})i))\cup HNInt(HNCl((A_{HN})i))$. Thus,$\cup_i\in\triangle(A_{HN})i\subseteq\cup_i\in\triangle HNCl(HNInt((A_{HN})i))\subseteq HNCl(\cup_{i\in\triangle}HNInt((A_{HN})i))$. Hence $\cup_{i\in\triangle}(A_{HN})i\subseteq HNCl(HNInt(\cup_{i\in\triangle}(A_{HN})i))$. Therefore, the union of a finite collection of HN-b- open sets is again a HN- b-open set in the HNTS X.

Remark 3.5 The intersection of any two HN-b-open sets need not be a HN-b-open sets.

Theorem 3.6 Let A_{HN} be a HNBO set in the HNTS X and suppose $A_{HN}\subseteq B_{HN}\subseteq HNCl(A_{HN})\cup HNInt(A_{HN})$. Then B_{HN} is HNBO set in X.

Proof: There exists a heptagonal neutrosophic open set HNO such that $HNO\subseteq A_{HN}\subseteq HNCl(HNO)\cup HNInt(HNO)$.
Since, $A_{HN}\subseteq B_{HN}$, $HNO\subseteq B_{HN}$.
But $HNCl(A_{HN})\cup HNInt(A_{HN})\subseteq HNCl(HNO)\cup HNInt(HNO)$.
And thus $B_{HN}\subseteq HNCl(HNO)\cup HNInt(HNO)$. Hence $HNO\subseteq B_{HN}\subseteq HNCl(HNO)\cup HNInt(HNO)$.
And B_{HN} is HNBO set in X.

Theorem 3.7: Every heptagonal neutrosophic open set in the HNTS X is a HNBO set in X.

Proof: Let A be a heptagonal neutrosophic open set in HNTS X. Then $A_{HN} = HNInt(A_{HN})$.
Also $HNInt(A_{HN})\subseteq HNCl(HNInt(A_{HN}))\cup HNInt(HNCl(A_{HN})$
This implies that $A_{HN}\subseteq HNCl(HNInt(A_{HN}))\cup HNInt(HNCl(A_{HN})$. Hence by Theorem 3.2, A_{HN} is a HNBO set in X.

4. Heptagonal neutrosophic b-interior in Heptagonal neutrosophic topological spaces

In this section, we introduce the heptagonal neutrosophic b-interior operator and their properties in the heptagonal neutrosophic topological space.

Definition 4.1: Let (X,τ) be a HNTS. Then for a heptagonal neutrosophic subset A_{HN} of X, the heptagonal neutrosophic b-interior of A_{HN} [HN-BInt (A_{HN}) for short] is the union of all heptagonal neutrosophic b-open sets of X contained in AHN. HN-BInt $(A_{HN}) = \cup\{B_{HN} : B_{HN}$is a HNBO set in X and $B_{HN}\subseteq A_{HN}\}$.

Proposition 4.2: Let (X,τ) be a HNTS.

Then for any heptagonal neutrosophic subsets A_{HN} and B_{HN} of a HNTS X we have

(i) HN-BInt $(A_{HN})\subseteq A_{HN}$
(ii) A_{HN} is HNBO set in X iff HN-BInt $(A_{HN}) = A_{HN}$
(iii) HN-BInt (HN-BInt (A_{HN})) = HN-BInt (A_{HN})
(iv) If $A_{HN}\subseteq B_{HN}$ then HN-BInt $(A_{HN})\subseteq$HN-BInt (B_{HN})
(v) HN-BInt($A_{HN}\cap B_{HN}$) = HN-BInt $(A_{HN})\cap$ HN-BInt (B_{HN})
(vi) HN-BInt $(A_{HN})\cup$ HN-BInt $(B_{HN})\subseteq$ HN-BInt $(A_{HN}\cup B_{HN})$

Proof:

(i) Follows from Definition 4.1.
HN-BInt $(A_{HN}) = \cup\{B_{HN} : B_{HN}$is a HNBO set in X and $B_{HN}\subseteq A_{HN}\}$. HN-BInt $(A_{HN})\subseteq A_{HN}$

(ii) Let A_{HN}be a HNBO set in X. Then $A_{HN}\subseteq$HN-BInt(A_{HN}).
By using (i) we get A_{HN} = BInt(A_{HN}).Conversely assume that A_{HN} = HN-BInt(A_{HN}). By using Definition 4.1,A_{HN} is NBO set in X.Thus (ii) is proved.

(iii) HN-BInt (HN-BInt (A_{HN})) = HN-BInt (A_{HN})
By using (ii)HN-BInt(HNBInt(A_{HN}))=HN-BInt(A_{HN}).
This proves (iii). Since $A_{HN}\subseteq B_{HN}$, by
using (i), HN-BInt$(A_{HN})\subseteq A_{HN}\subseteq B_{HN}$. That is HN-BInt$(A_{HN})\subseteq B_{HN}$. Thus (iii) is proved .

(iv) By (iii), HN-BInt(HN-SInt(A_{HN}))$\subseteq$HN-BInt(B_{HN}). Thus HN-BInt$(A_{HN})\subseteq$HN-SInt(B_{HN}).
Thus (iv) is proved.

(v) Since $A_{HN}\cap B_{HN}\subseteq A_{HN}$ and $A_{HN}\cap B_{HN}\subseteq B_{HN}$, by using (iv), HN-BInt $(A_{HN}\cap B_{HN})\subseteq$HN-BInt (A_{HN}) and HN-BInt$(A_{HN}\cap B_{HN})\subseteq$HN-BInt$(B_{HN})$. This implies that

HN-BInt(A_{HN}∩ B_{HN})⊆ HN-BInt(A_{HN})∩HN-BInt(B_{HN}) ---(1). By(i), HN-BInt(A_{HN})⊆A_{HN} and HN-BInt(B_{HN})⊆B_{HN}.

This implies that

HN-BInt(A_{HN})∩HN-BInt(B_{HN}) ⊆A_{HN}∩B_{HN}.

Now by (iv), HN-BInt ((HN-SInt(A_{HN})∩HN-SInt(B_{HN})) ⊆ HN-BInt(A_{HN}∩ B_{HN}). By (1), HN-BInt(HN-SInt (A_{HN}))∩HN-BInt(HN-SInt(B_{HN}))⊆HN-BInt(A_{HN}∩ B_{HN}). By (iii), HN-BInt(A_{HN})∩HN-BInt (B_{HN})⊆HN-BInt(A_{HN}∩ B_{HN}) (2).

From (1) and (2), HN-BInt (A_{HN}∩ B_{HN}) = HN-BInt(A_{HN})∩HN-BInt(B_{HN}).

Thus (v) is proved.

(vi) Since A_{HN}⊆A_{HN}∪B_{HN} and B_{HN}⊆ A_{HN}∪ B_{HN}, by (iv), HN-BInt (A_{HN}) ⊆ HN-BInt (A_{HN}∪ B_{HN}) and HN-BInt (B_{HN}) ⊆ HN-BInt (A_{HN}∪ B_{HN}). This implies that,

HN-BInt (A_{HN}) ∪ HN-BInt (B_{HN}) ⊆HN-BInt (A_{HN}∪ B_{HN}). Thus (vi) is proved.

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5. Heptagonal neutrosophic b-closure in heptagonal neutrosophic topological spaces

In this section, we introduce the heptagonal neutrosophic b-closure operator and its properties in the heptagonal neutrosophic topological space.

Definition 5.1: Let (X,) be a HNTS. Then for a heptagonal neutrosophic subset A_{HN} of X, the heptagonal neutrosophic b-closure of A_{HN} [HN-BCl (A_{HN}) for short] is the intersection of all heptagonal neutrosophic b-closed sets of X contained in A_{HN}. HN-BCl (A_{HN}) = ∪{ K_{HN} : K_{HN} is a HNBC set in X and A_{HN}⊆K_{HN}}.

Proposition 5.2: Let (X, τ) be a HNTS.

Then for any heptagonal neutrosophic subsets A_{HN} and B_{HN} of a HNTS X we have

(i) A_{HN}⊆ HN-BCl (A_{HN})

(ii) A_{HN} is HNBC set in X iff HN-BCl (A_{HN}) = $_{AHN}$

(iii) HN-BCl (HN-BCl (A_{HN})) = HN-BCl (A_{HN})

(iv) If A_{HN}⊆ B_{HN} then HN-BCl (A_{HN}) ⊆ HN-BCl (B_{HN})

(v) HN-BCl (A_{HN}∩ B_{HN}) ⊆HN-BCl (A_{HN}) ∩ HN-BCl (B_{HN})

(vi) HN-BCl (A_{HN}) ∪ HN-BCl (B_{HN}) = HN-BCl (A_{HN}∪ B_{HN})

Proof:

(i) Follows from Definition 5.1.

(ii) Let A_{HN}be a HNBC set in X. Then A_{HN} contains HN-BCl(A_{HN}). Now by using (i), we get A_{HN} = HN-BCl(A_{HN}). Conversely assume that A_{HN} = HN-BCl(A_{HN}). By using Definition 5.1, A_{HN} is a HNBC set in X. Thus (ii) is proved.

(iii) By using (ii), HN-BCl(HN-BCl(A_{HN})) = HN-BCl(A_{HN}). This (iii) is proved.

(iv) Since A_{HN}⊆B_{HN}, by using (i), B_{HN}⊆HN-BCl(B_{HN}) implies A_{HN}⊆HN-BCl(B_{HN}) . But HN-BCl(A_{HN}) is the smallest closed set containing A_{HN},

hence HN-BCl(A_{HN}) ⊆HN-BCl(B_{HN}). Thus (iv) is proved.

(v) Since A_{HN} ∩B_{HN}⊆A_{HN} and A_{HN}∩B_{HN}⊆B_{HN}, by using (iv), HN-BCl (A_{HN} ∩ B_{HN}) ⊆ HN-BCl (A_{HN}) and HN-BCl(A_{HN}∩B_{HN})⊆HN-BCl(B_{HN}).

This implies that HN-BCl(A_{HN}∩ B_{HN}) ⊆ HN-BCl(A_{HN})∩HN-BCl(B_{HN}). Thus (v) is proved

(vi) Since A_{HN}⊆A_{HN}∪B_{HN} and B_{HN}⊆A_{HN}∪ B_{HN}, by (iv), HN-BCl (A_{HN}) ⊆HN-BCl (A_{HN}∪ B_{HN}) and

HN-BCl (B_{HN}) ⊆ HN-BCl (A_{HN}∪ B_{HN}).

This implies that,

HN-BCl (A_{HN}) ∪ HN-BCl (B_{HN}) ⊆HN-BCl (A_{HN}∪ B_{HN}) ------ (1)

By(i), A_{HN}⊆ HN-BCl(A_{HN}) and B_{HN}⊆HN-BCl(B_{HN}).

This implies that A_{HN}∪B_{HN}⊆HN-BCl(A_{HN}) ∪ HN-BCl(B_{HN}).

Now by (iv), HN-BCl(A_{HN}∪ B_{HN})⊆HN-BCl ((HN-BCl(A_{HN})∪HN-BCl(B_{HN})). By (1), HN-BCl(A_{HN}∪ B_{HN})⊆HN-BCl(HN-BCl (A_{HN}))∪HN-BCl(HN-BCl(B_{HN})).

By (iii), HN-BCl(A_{HN}∪ B_{HN})⊆HN-BCl(A_{HN})∪HN-BCl (B_{HN})----- (2).

From (1) and (2), HN-BCl (A_{HN}∪ B_{HN}) = HN-BCl(A_{HN})∪HN-BCl(B_{HN}).

Thus (vi) is proved.

Proposition 5.4: Let (X, τ) be a HNTS. Then for any heptagonal neutrosophic subsets $A_{HN}$of a HNTS X, we have

(i) $(\text{HN-BInt}(A_{HN}))' = \text{HN-BCl}(A'_{HN})$

(ii) $(\text{HN-BCl}(A_{HN}))' = \text{HN-BInt}(A'_{HN})$

Proof:

(i) By definition 4.1, HN-BInt $(A_{HN}) = \cup \{ B_{HN} : B_{HN}$ is a HNBO set in X and $\mathbf{B}_{HN} \subseteq A_{HN}\}$ Taking the complement on both sides,

$(\text{HN-BInt}(A_{HN}))' = \cap \{ \mathbf{B}'_{HN} : \mathbf{B}'_{HN}$ is a HNBC set in X and $A'_{HN} \subseteq \mathbf{B}'_{HN}\}$

Now, replace $\mathbf{B}'_{HN}$ with K_{HN}, we get

$(\text{HN-}\mathbf{B}\text{Int}(A_{HN}))' = \cap \{K_{HN} : K_{HN}$is a HNBC set in X and $A'_{HN} \subseteq K_{HN}\}$ By definition 5.1,

$(\text{HN-}\mathbf{B}\text{Int}(A_{HN}))' = \text{HN-}\mathbf{B}\text{Cl}(A'_{HN})$. Thus (i) is proved.

(ii) From (i) for the HNS A'_{HN}

We write, $(\text{HN-}\mathbf{B}\text{Int}(A'_{HN}))' = \text{HN-}\mathbf{B}\text{Cl}(A_{HN})$

Taking the complement on both sides we get

$\text{HN-}\mathbf{B}\text{Int}(A'_{HN}) = (\text{HN-}\mathbf{B}\text{Cl}(A_{HN}))'$. Thus (ii) is proved.

6. Conclusion

The notion of heptagonal neutrosophic b-open sets and their characterization were presented and examined in this paper. It can also be expanded upon in the areas of quotient, continuous, and contra-continuous mappings. It is possible to investigate the set's homeomorphism, connectedness, and compactness in further detail.

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