IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

Structure of binary trees of 2-adic valuation for non-negative integers in the image of octagon

V. Pandichelvi[1], R. Vanaja [2]

[1]Assistant Professor, [2]Assistant Professor
[1]PG & Research Department of Mathematics,
Urumu Dhanalakshmi College, Trichy.
(Affiliated to Bharathidasan University)
[2]PG & Research Department of Mathematics,
Shrimati Indira Gandhi College, Trichy.
(Affiliated to Bharathidasan University)

Abstract : In this manuscript, the 2-adic valuation of octagonal numbers, sub sequences of non-negative integers in the presentation of octagonal shape which affords certain patterns and octa decagonal numbers are discussed. Also, binary trees of 2-adic valuation for the aforesaid sequences are monitored.

IndexTerms* - *Octagonal numbers, Octa decagonal numbers, prime factorization, 2-adic valuation, Binary tree.

1.INTRODUCTION

A set of numbers that can be set up into a rectangular octagon is represented by an octagonal number. The p-adic valuation of n, referred to as $v_p(n)$, is the exponent of the largest power of p that divides n for a prime integer p and $n \in N = \{1, 2, 3, \dots\}$. In [1-5], some properties of figurative numbers were discussed. Authors have examined the special characteristics of p $-$adic valuation of polygonal numbers in [6-8, 11-13]. In [9, 10], solutions to quadratic and cubic Diophantine equations were found.
In this manuscript, the 2-adic valuation of octagonal numbers, sub sequences of non-negative integers in the presentation of octagonal shape which delivers peculiar patterns and octa decagonal numbers are deliberated. Also, binary trees for 2-adic valuation for the abovementioned sequences are supervised.

2. DEPICTION OF NON-NEGATIVE INTEGERS IN THE SHAPE OF OCTAGON

Signify the patterns of non-negative integers in the shape of octagon as given in the following diagram.

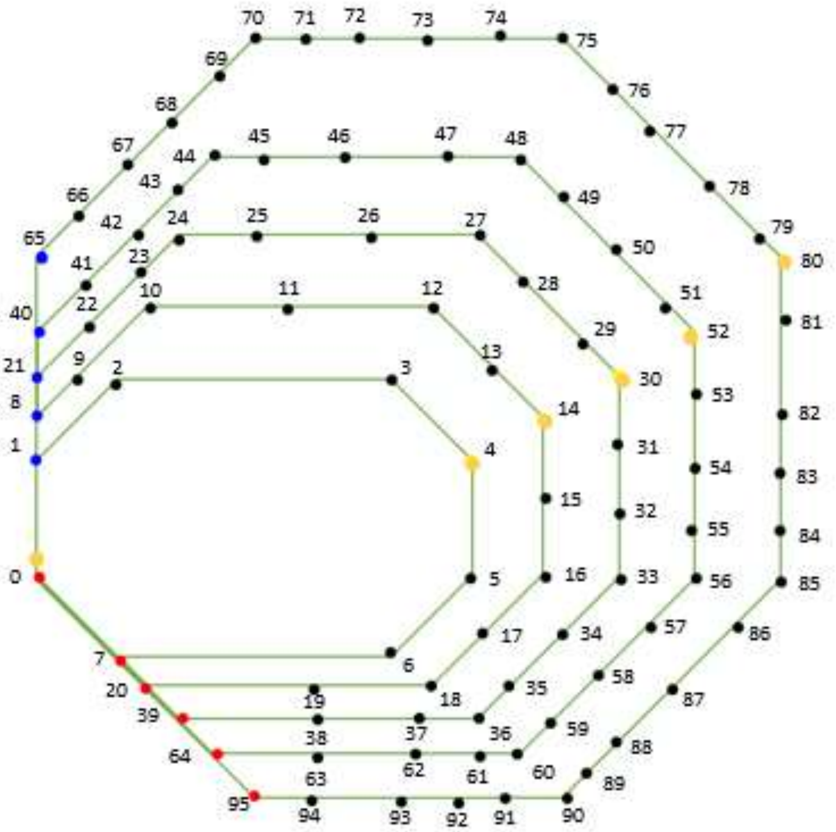

Fig. 2.1: Depiction of non-negative integers in the shape of octagon

The level-by-level procedure for the calculation of 2-adic valuation tree for explicit sequences of numbers marked in three dissimilar colours are established in sections 2.1 to 2.3.

2.1. Establishment of 2-adic valuation tree for octagonal numbers in blue colour

The octagonal numerals $1,8,21,40,65,\ldots$ are the ones specified in blue colour and it is represented by $B_1(n) = 3n^2 - 2n, n \in N$. It is detected that $\{0,3,0,3,0,5,\ldots.\}$ is the sequence of 2-adic valuations of $B_1(n)$.

Level 0:

Due to the fact that $3n^2 - 2n, n \in N$ comprises multiples of 2, the 2-adic valuation tree of the octagonal numbers are detached into two parts such as non-terminating and terminating vertices. Let us designate the non-terminating vertex with the notation $(*)$. For the left vertex, the corresponding numbers are even and of the form $2n$. Therefore, the reminder $c_0 = 0$ when $2n$ is divided by 2. The odd numbers corresponding to the right vertex follow the pattern $2n + 1$ and they do not divide by 2. Hence, $c_0 = 1$. $B_1(2n)$ and $B_1(2n+1)$ can be used to determine the 2-adic valuation of these vertices.

Now,

$V_2(B_1(2n)) = V_2\big(4(3n^2 - n)\big) = V_2(2^2) + V_2(3n^2 - n) = 2 + V_2(3n^2 - n) \geq 2$

$V_2(B_1(2n+1)) = V_2(12n^2 + 8n + 1) = 0$

As a result, the number with $2n + 1$ ends with a valuation of zero, and the number with $2n$ is separated into two vertices with a value of at least 2.

Level 1:

The number associated with a vertex for one branch of even number is $4n$ and $c_0 = 0$, $c_1 = 0$ while the vertex of another branch is $4n + 2$ and $c_0 = 0$, $c_1 = 1$

$V_2(B_1(4n)) = 3 + V_2(6n^2 - n) \geq 3$

$V_2(B_1(4n+2)) = 3 + V_2(6n^2 + 5n + 1) \geq 3$

Therefore, both vertices are non-terminating with a valuation of at least three at level 1.

Level 2:

At level 1, the even vertex is divided into two vertices. The left vertex is linked to $8n$ and $c_0 = c_1 = c_2 = 0$, whereas the right vertex is linked to $8n + 4$ and $c_0 = c_1 = 0$, $c_2 = 1$.

$V_2(B_1(8n)) = 4 + V_2(12n^2 - n) \geq 4$

$V_2(B_1(8n + 4)) = 3 + V_2(24n^2 + 22n + 5) = 3$

Therefore, the even vertex has a valuation of at least 4, which is non-terminating and the odd vertex has a valuation of 3 and comes to an end.

First-level odd vertex is divided into two vertices. The left and right vertices are represented by $8n + 2$ and $8n + 6$ respectively.

$V_2(B_1(8n + 2)) = 3 + V_2(24n^2 + 10n + 1) = 3$

$V_2(B_1(8n + 6)) = 4 + V_2(12n^2 + 17n + 6) \geq 4$

If $c_0 = 0, c_1 = 1, c_2 = 0$, the vertex is terminating with a valuation of 4.

On the other hand, if $c_0 = 0, c_1 = 1, c_2 = 1$, the vertex is non-terminating with a valuation of at least 4.

Level 3:

The vertex is break up into two vertices with $16n$ and $16n + 8$ if $c_0 = c_1 = c_2 = 0$.

$V_2(B_1(16n)) = 5 + V_2(24n^2 - n) \geq 5$

$V_2(B_1(16n + 8)) = 4 + V_2(48n^2 + 46n + 11) = 4$.

The vertex is non-terminating with a valuation of at least 5 if $c_0 = c_1 = c_2 = c_3 = 0$ and the other vertex is terminating with a valuation of 4 if $c_0 = c_1 = c_2 = 0, c_3 = 1$.

The non-terminating vertex is fragmented into two vertices with $16n + 6$ and $16n + 14$ if $c_0 = 0, c_1 = 1, c_2 = 1$.

$V_2(B_1(16n + 6)) = 5 + V_2(24n^2 + 17n + 3) \geq 5$

$V_2(B_1(16n + 14)) = 4 + V_2(48n^2 + 82n + 35) = 4$

If $c_0 = 0, c_1 = 1, c_2 = 1, c_3 = 0$, then the vertex does not terminate and has a valuation of at least 5 and if $c_0 = 0, c_1 = 1, c_2 = 1, c_3 = 1$, then the vertex is terminating with a valuation of 4.

By repeating the previously described procedure, this sequence has an endless double- branching 2-adic valuation tree.

The 2-adic valuation tree of the sequence $B_1(n)$ is portrayed in Figure 2.1.

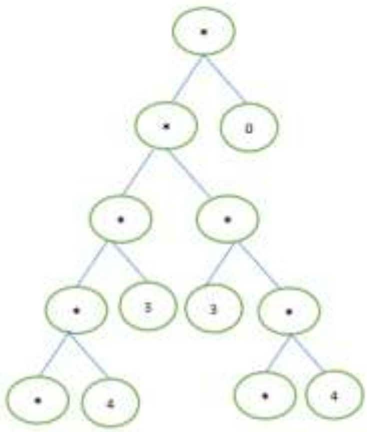

Fig 2.2: 2-adic valuation tree of $B_1(n)$

The characteristics of the 2-adic valuation of $B_1(n)$ are illustrated by the following theorem.

Theorem 2.1

Let v_k be the node at level k. The properties of 2-adic valuation tree of $B_1(n)$ are as follows.

(i) Suppose $k = 0$. When $c_k = 0$, the vertex beneath v_k is a non-terminating vertex with valuation of at least 2 and when $c_k = 1$, the vertex terminates with zero valuation.

(ii) If $k = 1$, then the two vertices detached from v_k are non-terminating and have a valuation of at least three.

(iii) If $k > 1$ then v_k is divided into two vertices at the $(k+1)^{\text{th}}$ level. The one with $c_k = 1$ ends with a valuation of $k+1$, while the other with $c_k = 0$ does not end with a valuation of at least $k+2$ along with $c_0 = 0, c_1 = 0$.

(iv) If $k > 1$ and v_k be a nonterminating vertex with $c_0 = 0, c_1 = 1$ then v_k is split up into two vertices at the $(k+1)^{th}$level. The one terminates with valuation $k+1$ and the other has valuation of at least $k+2$.

Proof

(i) Let $B_1(n) = 3n^2 - 2n, n \in N$ and $k = 0$.

Since it contains multiples of 2, it is divided into two vertices with $2n$ and $2n+1$.

$V_2(B_1(2n)) = 2 + V_2(3n^2 - n) \geq 2$

$V_2(B_1(2n+1)) = V_2(12n^2 + 8n + 1) = 0$.

The vertex is non-terminating with a valuation of at least 2 if $c_0 = 0$ and the vertex is terminating with a valuation zero if $c_0 = 1$.

(ii) Assume $k = 1$.

At level 0, the even vertex is subdivided into two vertices, namely $4n$ and $4n+2$.

Both the vertices are nonterminating and possess a valuation of at least 3 since $V_2(B_1(4n)) = 3 + V_2(6n^2 - n) \geq 3$ and $V_2(B_1(4n+2)) = 8 + V_2(6n^2 + 5n + 1) \geq 3$

(iii) Assume v_k is a nonterminating vertex that descends from v_1 with $c_1, c_0 = 0$ and $k > 1$.

To demonstrate the outcome by mathematical induction on k.

Let $k = 2$

The second level even vertex is divided into two vertices with $8n$ and $8n+4$.

$V_2(B_1(8n)) = 4 + V_2(12n^2 - n) \geq 4$

$V_2(B_1(8n+4)) = 3 + V_2(24n^2 + 22n + 5) = 3$

The one with $c_2 = 1$ ends with a valuation of 3 while the other with $c_2 = 0$ does not end with a valuation of at least 4.

Therefore, the result is true for $k = 2$.

Assume $k = 3$

Level 3 divides the even vertex into two vertices namely $16n, 16n+8$.

$V_2(B_1(16n)) = 5 + V_2(24n^2 - n) \geq 5$

$V_2(B_1(16n+8)) = 4 + V_2(48n^2 + 46n + 11) = 4$

The vertex with $c_3 = 0$ is non-terminating and has a value of at least 5 while another vertex with $c_3 = 1$ terminates with valuation of 4.

The result remains true for $k = 3$.

Assume that the result is true for all $k = m$.

To demonstrate that the result holds up when $k = m+1$.

Then numbers for v_m have the form $2^m n + d_{m-1}$ and the numbers connected with vertices at level $m+1$ have the form $v_{m+1} = 2^{m+1}n + 2^m c_m + d_{m-1}$ where $d_{m-1} = 2^{m-1}c_{m-1} + \cdots + 2c_1 + c_0 = 0$. So $v_{m+1} = 2^{m+1}n + 2^m c_m$

Using the induction hypothesis, $B_1(d_{m-1}) = 3{d_{m-1}}^2 - 2d_{m-1} \equiv 0(mod\ 2^{m+1})$, implying that $V_2(B_1(d_{m-1})) \geq m+1$.

Note that,

$B_1(v_{m+1}) = 3{v_{m+1}}^2 - 2v_{m+1} \equiv 0(mod\ 2^{m+2}) \Longrightarrow c_m \equiv 0(mod\ 2)$

The vertex with $c_m = 1 \not\equiv 0(mod\ 2)$ terminates with a valuation of $k+1$ and the other with $c_m \equiv 0(mod\ 2)$ does not terminate with a valuation of at least $k+2$.

Subsequently, the result is valid for all $k > 1$.

(iv) Let $k > 1$ and v_k be a nonterminating vertex that descends from v_1 with $c_0 = 0$ and $c_1 = 1$. To establish the result by using mathematical induction for $k > 1$.

Assume that $k = 2$.

The left and right vertices of second level odd vertex are represented by $8n + 2$ and $8n + 6$ respectively.

$V_2(B_1(8n + 2)) = 3 + V_2(24n^2 + 10n + 1) = 3$

$V_2(B_1(8n + 6)) = 4 + V_2(12n^2 + 17n + 6) \geq 4$

If $c_0 = 0, c_1 = 1$ and $c_2 = 0$, then the vertex ends with a valuation of 4.

On the other hand, if $c_0 = 0, c_1 = 1, c_2 = 1$, then the vertex has a minimum valuation of 4 and it is nonterminating.

Thus, the result is true for $k = 2$.

Let $k = 3$

If $c_0 = 0, c_1 = 1, c_2 = 1$, then the vertex is separated into two vertices with $16n + 6$ and $16n + 14$.

$V_2(B_1(16n + 6)) = 5 + V_2(24n^2 + 17n + 3) \geq 5$

$V_2(B_1(16n + 14)) = 4 + V_2(48n^2 + 82n + 35) = 4$

If $c_0 = 0, c_1 = 1, c_2 = 1, c_3 = 0$, then the vertex does not terminate and has a valuation of at least 5. If $c_0 = 0, c_1 = 1$, $c_2 = 1, c_3 = 1$, then the vertex terminates with a valuation of 4.

Thus, the result is true for $k = 3$.

Suppose that the result is valid for all $k = l$.

For the evidence of the result, let $k = l + 1$.

Then v_l has numbers of the form $2^l n + d_{l-1}$ and the numbers associated with vertices at level $l + 1$ have the form $v_{l+1} = 2^{l+1}n + 2^l c_l + d_{l-1}$ where $d_{l-1} = 2^{l-1}c_{l-1} + \cdots + 2$. So $v_{l+1} = 2^{l+1}n + 2^l c_l + d_{l-1}$.

By induction hypothesis, $B_1(d_{l-1}) = 3{d_{l-1}}^2 - 2d_{l-1} \equiv 0(mod\ 2^{l+1})$ so that $V_2(B_1(d_{l-1})) \geq l + 1$.

Also,

$B_1(v_{l+1}) = 3{v_{l+1}}^2 - 2v_{l+1} \equiv 0(mod\ 2^{l+2}) \Longrightarrow c_l \equiv m(mod\ 2)$.

If $c_l \neq m \not\equiv 0(mod\ 2)$, the vertex terminates with $l + 1$ and the other has at least $l + 2$.

Therefore, the result is true for $k = l + 1$.

Hence the result is true for all $k > 1$.

2.2. Evaluation of 2-adic valuation tree for the yellow-coloured sequence

The 2-adic valuation tree for the sequence $\{0,4,14,30,52,80,\ldots\}$ which is represented by a yellow colour in an octagonal shape is discussed in this section. This sequence has the form $Y_1(m) = 3m^2 - 5m + 2, m \in N$. It is identified that the 2-adic valuation of this sequence belongs to the collection $\{0,2,1,1,2,4,\ldots\}$.

Follow the steps given in section 2.1, the binary 2-adic tree of $Y_1(m)$ is shown in Fig.2.3.

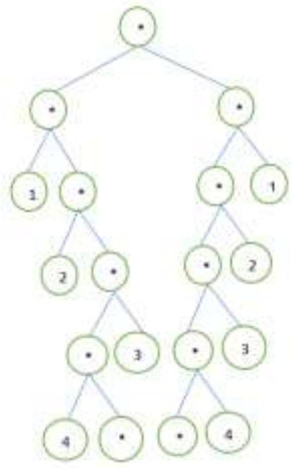

Fig 2.3: 2-adic valuation tree of $Y_1(m)$

The characteristics of the 2-adic valuation of $Y_1(m)$ are illustrated by the following theorem.

Theorem 2.2

Let v_k be the nonterminating node at level k. The 2-adic valuation tree of $Y_1(m) = 3m^2 - 5m + 2, m \in N$ has the following characteristics.

(i) If $k = 0$, then the both vertices are non-terminating with a valuation of minimum 1.

(ii) Assume that $c_0 = 0$ and that v_k is a non-terminating vertex downward form v_0 for $k > 1$. At $(k+1)^{th}$ level, v_k is divided into two vertices. One ends with a valuation of k, while the other does not end with a valuation of at least $k+1$.

(iii) Let $k > 1$ and v_k be a non-terminating vertex from v_0 with $c_0 = 1$. The one with $c_k = 1$ terminates with valuation k and the other with $c_k = 0$ has valuation of $k+1$.

Proof

(i) Choose $k = 0$.

The starting node is partitioned into two vertices $2m$ and $2m+1$, respectively, because $Y_1(m)$ consists multiples of 2.

$Y_1(2m) = 2(6m^2 - 5m + 1) \Longrightarrow V_2(T_{8,2m}) \geq 1$ and $Y_1(2m+1) = 2(6m^2 + m) \Longrightarrow V_2(T_{8,2m+1}) \geq 1$

As a result, both vertices are non-terminating and have a valuation of at least 1.

(ii) Let $k > 1$ and v_k be a nonterminating vertex descending form $v_0$with $c_0 = 0$.

According to the argument of (iii) of theorem 2.1, $5c_k \equiv t\ (mod\ 2)$. Therefore, the vertex that satisfies the condition $5c_k \not\equiv t\ (mod\ 2)$ terminates with a value of k and the other is separated into two vertices with a valuation of at least $k+1$.

(iii) If $k > 1$ and $c_0 = 1$, then v_k be a non-terminating vertex from v_0.

As in the proof of theorem 2.1, $c_k \equiv 0\ (mod\ 2)$.

If $5c_k \equiv 0\ (mod\ 2)$, then the vertex is terminating with a valuation of k, whereas the other vertex is non-terminating with valuation of at least $k+1$.

2.3 Description of 2-adic valuation of red coloured sequence

This section discusses the 2-adic valuation tree for the red-coloured sequence $\{0,7,20,39,64,\dots\}$ in Fig. 2.1. Consider this sequence as $R_1(l) = 3l^2 - 2l - 1, l \in N$. This sequence has the 2-adic valuation $\{0,0,4,0,6,\dots\}$. Following the procedure as described in section 2.1, the binary tree for 2-adic valuation of $R_1(l)$ is depicted in fig.2.4.

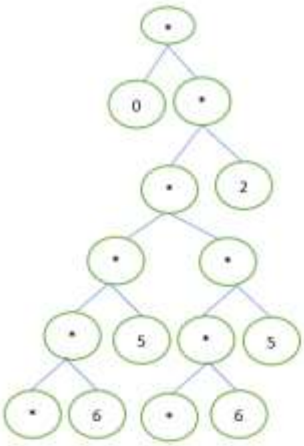

Fig 2.4: 2-adic valuation tree of $R_1(l)$

The subsequent theorem explains the properties of the 2-adic valuation of $R_1(l)$

Theorem 2.3

Assume that v_k indicates the nonterminating node at level k. The 2-adic tree of $R_1(l) = 3l^2 - 2l + 1, l \in N$ holds the following properties:

(i) Let $k = 0$. The one with $c_k = 0$ terminates with a valuation of k while the other is non-terminating and has a valuation of $2k + 2$.

(ii) If $k = 1$, the vertex with $c_k = 1$ end with a value of $k + 1$and the other vertex has a valuation of at least $2k + 2$ that does not end.

(iii) Let $k = 2$, the both the vertices are nonterminating with a valuation of at least $k + 3$

(iv) Let $k > 2$ and v_k be a nonterminating vertex from v_2 with $c_0 = 1, c_1 = 0, c_2 = 0$. The vertex along with $c_k = 1$ comes to an end with a valuation of k+2 and the other does not end with a valuation of $k + 3$.

(v) Assume $k > 2$ and v_k is a nonterminating vertex beneath from v_2 with $c_0 = 1, c_1 = 0, c_2 = 1$. The vertex is partitioned into two vertices at $(k+1)^{th}$level. The one terminates with a valuation of $k + 2$ and other with valuation of at least $k + 3$.

Proof

(i) Let $k = 0$.

The initial node is divided into two vertices with $2l$ and $2l + 1$, because $R_1(l)$ contains multiples of 2.

Here,

$R_1(2l) = 12l^2 - 2l + 1 \Longrightarrow V_2(R_1(2l)) = 0$ and $R_1(2l + 1) = 4(3l^2 + 2l) \Longrightarrow V_2(R_1(2l + 1)) \geq 2$

Hence, the vertex with $c_k = 0$ with a zero value and the other does not end with a minimum value of 2.

(ii) Let $k = 1$ with $c_0 = 1$.

Also,

$R_1(4l + 1) = 16(3l^2 + l) \Longrightarrow V_2(\, R_1(4l + 1)) \geq 4$ and $R_1(4l + 3) = 4(12l^2 + 16l + 5) \Longrightarrow V_2(\, R_1(4l + 3)) = 2$

The vertex with $c_0 = 1,\ c_1 = 1$ terminates with a value of $k + 1$ whereas the other vertex does not terminate and has a valuation of at least $2k + 2$.

(iii) Let $k = 2$ with $c_0 = 1, c_1 = 0$.

Note that

$R_1(8l+1) = 32(6l^2 + l) \Longrightarrow V_2(\,R_1(8l+1)) \geq 5$ and $R_1(8l+5) = 32(6l^2 + 7l + 2) \Longrightarrow V_2(\,R_1(8l+5)) \geq 5$

Hence, both vertices are nonterminating with a valuation of at least $k + 3$.

(iv) Let $k > 2$ with $c_0 = 1, c_1 = 0, c_2 = 0$.

Here $c_i = 0$ for $i = 3,4, \dots, k-1$.

Therefore, $d_{k-1} = 1$.

At level $k+1$, $N_k = 2^{k+1}l + 2^k c_k + 1$.

As in theorem 2.1(iii), $c_k \equiv 0\ (mod\ 2)$.

If $c_k = 1 \not\equiv 0\ (mod\ 2)$, then the vertex terminates with a value of $k+2$ and other vertex does not terminate with a value of at least $k+3$.

(v) Consider $k > 2$ with $c_0 = 1, c_1 = 0, c_2 = 1$

The numbers connected with the vertex at level $k+1$ underneath the vertex v_k have the form $N_k = 2^{k+1}.l + 2^k c_k + d_{k-1}$, and v_k is then of the form $2^k l + d_{k-1}$ where $d_{k-1} = 2^{k-1}c_{k-1} + \dots + 2c_1 + c_0 = 2^{k-1}c_{k-1} + \dots + 4 + 1 = 2^{k-1}c_{k-1} + \dots + 5$.

$3d_{k-1} - 1 = 2p$, where p be any whole number.

Also, $c_k \equiv t\ (mod\ 2)$, as demonstrated in the proof of theorem 2.1(iv).

Accordingly, if $c_k \not\equiv t\ (mod\ 2)$ ends with a valuation of $k+2$ and the other does not end, that has a valuation of at least $k+3$.

3. Construction of 2-adic valuation tree for octa decagonal numbers

The sequence of octa decagonal numbers $\{1,18,51,100,165,246,343,456, \dots\}$ generates the quadratic polynomial $T_{18,n} = 8n^2 - 7n, n \in N$ which has a 2-adic valuation of $\{0,1,0,2,0,1,0,3, \dots\}$. The 2-adic valuation tree for this sequence is constructed from the root vertex at level 0 and the numbers corresponding to it take the form n, $n \in N$.

Since the expression $8n^2 - 7n, n \in N$ contains multiples of 2, $V_2(8n^2 - 2n) \geq 1$.

As a result, the root vertex is non-terminating and separated into two vertices. For the left vertex, the corresponding numbers are even and of type $2k$, while the odd integers of pattern $2k+1$ correspond to the right vertex. These vertices have 2-adic values as $V_2(\,T_{18,2k})$ and $V_2(T_{18,2k+1})$.

Now,

$V_2(8(2k)^2 - 7(2k)) = V_2(2(16k^2 - 7k)) = 1 + V_2(16k^2 - 7k) \geq 1$

The even vertex is non-terminating and has a valuation of at least one since $16k^2 - 7k$ contains multiplies of 2.

Also,

$V_2(8(2k+1)^2 - 7(2k+1)) = V_2(32k^2 + 18k + 1) = 0$ since $32k^2 + 18k + 1, k \in N$ is odd.

Therefore, the odd vertex terminates with a valuation of zero.

At level 2, the even vertex of the tree at level 1 breaks up into two branches with vertices of forms $4l$ and $4l+2$.

Now, $V_2(8(4l)^2 - 7(4l)) = V_2(128l^2 - 28l) \geq 2$ and

$V_2(8(4l+2)^2 - 7(4l+2)) = V_2(128l^2 + 100l + 18) = 1$

Thus, the vertex comprising the numbers $4l$ is not terminating with a valuation of at least 2 while the vertex having the numbers $4l+2$ is terminating with a valuation of 1.

Consequently, two vertices with the numbers $8m$ and $8m+4$ split off from the even vertex.

$V_2(8(8m)^2 - 7(8m)) = V_2(512m^2 - 56m) \geq 3$ and

$V_2(8(8m+4)^2 - 7(8m+4)) = V_2(512m^2 + 456m + 100) = 2$

The vertex with numbers $8m$ does not terminate with a valuation of at least 3 whereas the vertex with numbers $8m + 4$ terminates with a valuation of 2.

The 2-adic value tree of the octa decagonal numbers is given in Fig. 3.1.

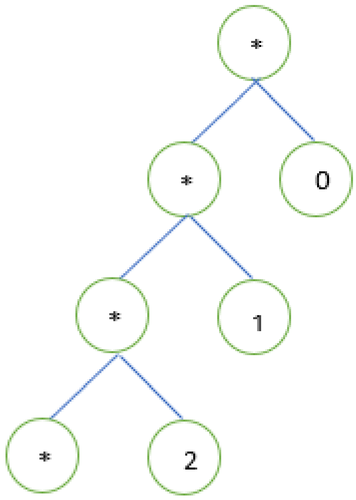

Fig 3.1: 2-adic valuation tree of octa decagonal number

The following theorem describes the nature of the 2-adic valuation of Octa decagonal number.

Theorem 3.1

In the valuation tree of $V_2(8n^2 - 7n), n \in N$, let v_k be a non-terminating node at the k^{th} level with $k \geq 1$. Then v_k is separated into two vertices at the $(k+1)^{th}$ level. One terminates with a valuation of k when $e_k = 1$. The other does not terminate and has valuation of at least $(k+1)$ when $e_k = 0$.

Proof

Let $T_{18,n} = 8n^2 - 7n, n \in N$

Since $T_{18,2n+1} = 32n^2 + 18n + 1$ is odd, $V_2(T_{18,2n+1}) = 0$.

Therefore, the odd branches terminate with a zero valuation.

The numbers corresponding to the vertex v_k in the even branch have the form $2^k n + f_{k-1}$ and the numbers corresponding to the vertices of the next level underneath v_k have the form $N_k = 2^{k+1}n + 2^k e_k + f_{k-1}$ where $f_{k-1} = 2^{k-1}e_k + \cdots + 2e_1 + e_0$.

On the left branches, $f_{k-1} = 0$

$v_k = 2^k n$ and $N_k = 2^{k+1}n + 2^k e_k$

By the induction hypothesis that $f(f_{k-1}) = 8{f_{k-1}}^2 - 7f_{k-1} \equiv 0(mod\ 2^k)$, $V_2(8{f_{k-1}}^2 - 7f_{k-1}) \geq k$.

Now,

$f(N_k) = 8{N_k}^2 - 7N_k \equiv 0(mod\ 2^{k+1})$

$\Rightarrow -7.2^k e_k \equiv 0\ (mod\ 2^{k+1})$

$\Rightarrow -7e_k \equiv 0\ (mod\ 2)$

If $e_k = 1$, then $-7e_k = -7 \not\equiv 0(mod\ 2)$. Therefore, the vertex v_k with $e_k = 1$ terminates with a valuation of k.

On the other hand, if $e_k = 0$, then $-7e_k \equiv 0\ (mod\ 2)$. Thus, the vertex v_k with $e_k = 0$ has the valuation of at least $k + 1$ and it is non-terminating vertex.

4. Conclusion

In the manuscript, binary trees of 2-adic valuation for octagonal numbers, sub sequences of non-negative integers in the appearance of octagonal shape which distributes particular forms and octa decagonal numbers are discussed. In this manner, one can try to draw p −adic valuation trees for numerous sequences for the prime $p > 3$.

REFERENCES

[1] John,H., and Richard K. Conway. 1995. “Guy. The Book of Numbers.” Springer Verlag, New York.

[2] Yan, Song Y. 2002. “Elementary number theory.” Number Theory for Computing. Springer, Berlin, Heidelberg, 1-172.

[3] Ivan Niven, Hibert S, Zuckerman and Huge L, Montgomery. 2004. “An Introduction to the Theory of Numbers.” John Wiley & Sons Inc., Fifth Edn.

[4] Vazzana, Anthony, Martin Erickson, and David Garth. 2007. “Introduction to number theory.” Chapman and Hall/CRC.

[5] Weil, Andre. 2013. “Basic number theory.” Springer Science & Business Media, 144.

[6] Sanna, Carlo. 2016. “On the p-adic valuation of harmonic numbers.” Journal of Number Theory, 166, 41-46.

[7] Sanna, Carlo. 2016. “The p-adic valuation of Lucas sequences.” Fibonacci Quart, 54 (2), 118-124.

[8] Boultinghouse, Will, et al. 2020. “2-adic Valuations of Quadratic Sequences.” arXiv preprint arXiv:2012.10332, 2020.

[9] Kozhushkina, Olena, et al. 2020. “The valuation tree for $n^2 + 7$.” SCIENTIA Series A: Mathematical Sciences, 30, 91-102.

[10] Maila Brucal-Hallare, Eva G. Goedhart, Ryan Max Riley, Vaishavi Sharma and Bianca hompson. 2021. “Solving Quadratic and Cubic Diophantine Equations using 2-adic Valuation Trees.” arXiv preprint arXiv:2105.03352.

[11] Singh, S. S., A. Lalchhuangliana, and P. K. Saikia. 2021. “On the p-adic Valuations of Stirling numbers of the second kind.” Contemporary Mathematics, 24-36.

[12] Noguera, Maribel del Carmen Dıaz. 2022. “p-adic valuation and linear forms in Fibonacci-like recurrence sequences.”.

[13] Young, Paul Thomas. 2022. “On the 2-Adic Valuation of Generalized Fibonacci Sequences.” Integers: Electronic Journal of Combinatorial Number Theory, 22.