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ON FUZZY NEUTROSOPHIC SEMI−OPEN(CLOSED) SETS IN FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract : In this paper is to define the fuzzy neutrosophic semi−open sets (closed sets) in fuzzy neutrosophic topological spaces. Also, we discuss some of its characterizations of fuzzy neutrosophic semi−open sets (closed sets) and fuzzy neutrosophic semi−interior(closure) are studied. Several examples are given to illustrate the concepts.

Keywords: Fuzzy neutrosophic semi−open set, fuzzy neutrosophic semi−closed set, fuzzy neutrosophic semi−interior, fuzzy neutrosophic semi−closure.

1.INTRODUCTION

The concept of fuzzy sets was first introduced by Lofti A. Zadeh [11] in 1965, in which he proposed that the traditional notion of set, in which an element is either a member or not a member, should be generalized to allow for degree of membership. This led to the development of fuzzy set theory, which has since been applied to a wide range of fields, including control theory, image processing, and artificial intelligence.

Neutrosophic sets were first introduced by Florentin Smarandache [7] in 1955, which is a generalization of the concepts of fuzzy sets and intuitionistic fuzzy sets. Neutrosophic sets allow for consideration of indeterminacy, neutrality, and inconsistency in a given space.In 2017, Veereswari [10] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields. In this paper, we introduce the concept of fuzzy neutrosophic semi-open (semi-closed) and fuzzy neutrosophic semi-interior(semi-closure) are introduced and several characterizations with examples are studied in fuzzy neutrosophic topological spaces.

2.PRELIMINARIES

Throughout the present paper, X denote the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted by $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic open set $(fnOS)$ if $A_N \leq fn\ (((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set $(fnCS)$.

Definition 2.1 [3]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ where $T, I, F: X \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [3]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.3 [3]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$
$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

Definition 2.4 [3]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [3]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0$, $I_A(x) = 0$, $F_A(x) = 1$ *for all* $x \in X$. It is denoted by 0_N.

Definition 2.6 [3]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x\ \in X$. It is denoted by 1_N.

Definition 2.7 [3]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$$A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle \text{ where } T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [8]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

$(i) 0_N, 1_N \in \tau$

$(ii) A_1 \cap A_2 \in \tau\ for\ any\ A_1, A_2 \in \tau$ $(iii)\ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i: i \in J\}\ \in \tau$

Satisfying the following axioms.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [8]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [8]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup \{G: G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$$
$$cl(A) = \cap \{G: G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$$

3. Fuzzy neutrosophic semi−open sets (closed sets) in fuzzy neutrosophic topological spaces.

In this section, we introduce the concept of fuzzy neutrosophic semi−open set (closed set) and also discussed their characterizations.

Definition 3.1:

Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N). Then A_N is said to be fuzzy neutrosophic semi−open set in (X, τ_N) if there exists a fuzzy neutrosophic open set such that $fnO \leq A_N \leq fncl(fnO)$.

The following theorem is the characterization of fuzzy neutrosophic semi−open set in fuzzy neutrosophic topological space.

Theorem 3.2

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi−open set if only if $A_N \leq ((A_N)^+)^-$.

Proof:

Sufficiency: Let $A_N \leq ((A_N)^+)^-$. Then for $fnO = (A_N)^+$, we have $fnO \leq A_N \leq fncl\ (fnO)$. *Necessity:* Let A_N be a fuzzy neutrosophic semi−open set in (X, τ_N). Then $fnO \leq A_N \leq fncl\ (fnO)$ for some fuzzy neutrosophic semi−open set A_N. But $fnO = (A_N)^+$ and thus $fncl\ (fnO) \leq ((A_N)^+)^-$. Hence $A_N \leq fncl\ (fnO) \leq ((A_N)^+)^-$.

Theorem 3.3:

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then the union of two fuzzy neutrosophic semi−open sets is a fuzzy neutrosophic semi−open set in the fuzzy neutrosophic topological space (X, τ_N).

Proof:

Let A_N and B_N are fuzzy neutrosophic semi−open sets in (X, τ_N). Then $A_N \leq ((A_N)^+)^-$ and $B_N \leq ((B_N)^+)^-$. Therefore $A_N \vee B_N \leq ((A_N)^+)^- \vee ((B_N)^+)^- = ((A_N)^+ \vee (B_N)^+)^- \leq ((A_N \vee B_N)^+)^-$. By proposition 3.11. hence $A_N \vee B_N$ is fuzzy neutrosophic semi−open set in (X, τ_N).

Theorem 3.4:

Let (X, τ_N) be a fuzzy neutrosophic topological space. If $\{A_{N_i}\}_{i \in \triangle}$ is a collection of fuzzy neutrosophic semi−open sets in a fuzzy neutrosophic topological space (X, τ_N). Then $\vee_{i \in \triangle} A_{N_i}$ is fuzzy neutrosophic semi−open set in (X, τ_N).

Proof:

For each $i \in \triangle$, we have a fuzzy neutrosophic semi−open set fnO_i such that $fnO_i \leq A_{N_i} \leq fncl\ (fnO_i)$.

Then $\vee_{i \in \triangle} fnO_i \leq \vee_{i \in \triangle} A_{N_i} \leq \vee_{i \in \triangle} fncl(fnO_i) \leq fncl(\vee_{i \in \triangle} fnO_i)$. Hence let $fnO = \vee_{i \in \triangle} fnO_i$.

Remark 3.5

The intersection of any two fuzzy neutrosophic semi−open sets need not be a fuzzy neutrosophic semi−open set in (X, τ_N) as shown by the following example.

Example 3.6:

Let $X = \{a, b, c\}$ and

$A_N = \{\langle a, 0.5, 0.3, 0.4\rangle, \langle b, 0.5, 0.2, 0.6\rangle, \langle c, 0.4, 0.1, 0.4\rangle\}$

$B_N = \{\langle a, 0.6, 0.1, 0.5\rangle, \langle b, 0.5, 0.8, 0.1\rangle, \langle c, 0.3, 0.2, 0.2\rangle\}$

$C_N = \{\langle a, 0.2, 0.4, 0.8\rangle, \langle b, 0.4, 0.2, 0.4\rangle, \langle c, 0.4, 0.2, 0.3\rangle\}$

Then $\tau_N = \{0_N, A_N, B_N, C_N, 1_N\}$ is fuzzy neutrosophic topological space on (X, τ_N). Now, we define the two fuzzy neutrosophic semi−open sets as follows:

$E_N = \{\langle a, 0.5, 0.4, 0.8\rangle, \langle b, 0.5, 0.3, 0.5\rangle, \langle c, 0.6, 0.2, 0.4\rangle\}$ and

$G_N = \{\langle a, 0.5, 0.3, 0.5\rangle, \langle b, 0.5, 0.3, 0.7\rangle, \langle c, 0.4, 0.2, 0.4\rangle\}$. Here $(E_N)^+ = C_N$ and $((E_N)^+)^- = 1$ and $(F_N)^+ = A_N$ and $((F_N)^+)^- = 1$.

But $E_N \wedge F_N = \{\langle a, 0.5,0.3,0.5\rangle, \langle b, 0.5,0.3,0.5\rangle, \langle c, 0.4,0.2,0.4\rangle\}$ is not a fuzzy neutrosophic semi−open set in (X, τ_N).

Theorem 3.7:

Let A_N be a fuzzy neutrosophic semi−open in the fuzzy neutrosophic topological space (X, τ_N) and suppose $A_N \leq B_N \leq (A_N)^-$. Then B_N is a fuzzy neutrosophic semi−open set in (X, τ_N).

Proof:

There exists a fuzzy neutrosophic open set fnO such that $fnO \leq A_N \leq fn(fnO)^-$. Then $\leq B_N$. But $(A_N)^- \leq fn(fnO)^-$ and thus $B_N \leq fn(fnO)^-$. Hence $fnO \leq B_N \leq fn(fnO)^-$ and B_N is a fuzzy neutrosophic semi−open set in (X, τ_N).

Theorem 3.8:

Every fuzzy neutrosophic open set in the fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi−open set in (X, τ_N).

Proof:

Let A_N be fuzzy neutrosophic open set in fuzzy neutrosophic topological space (X, τ_N). Then $A_N \leq (A_N)^+$. Also $(A_N)^+ \leq ((A_N)^+)^-$. This implies that $A_N \leq ((A_N)^+)^-$. Hence by theorem 3.2, A_N is a fuzzy neutrosophic semi−open set in (X, τ_N).

Remark 3.9:

In a fuzzy neutrosophic topological space, every fuzzy neutrosophic semi−open set in (X, τ_N) is need not to be fuzzy neutrosophic open set in (X, τ_N) as shown by the example.

Example 3.6:

Let $X = \{a, b, c\}$ and with $\tau_N = \{0_N, A_N, B_N, C_N, D_N, E_N, F_N, 1_N\}$. Some of the fuzzy neutrosophic semi−open sets are

$A_N = \{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0.3,0.1,0.2\rangle, \langle c, 0.8,0.6,0.9\rangle\}$

$B_N = \{\langle a, 0.4,0.2,0.5\rangle, \langle b, 0.2,0.1,0.1\rangle, \langle c, 0.8,0.5,0.6\rangle\}$

$C_N = \{\langle a, 0.1,0.6,0.5\rangle, \langle b, 0.3,0.1,0.4\rangle, \langle c, 0.5,0.9,0.8\rangle\}$

$D_N = \{\langle a, 0.4,0.5,0.3\rangle, \langle b, 0.2,0.1,0.6\rangle, \langle c, 0.5,0.8,0.7\rangle\}$

$E_N = \{\langle a, 0.6,0.5,0.1\rangle, \langle b, 0.1,0.6,0.4\rangle, \langle c, 0.8,0.5,0.9\rangle\}$

$F_N = \{\langle a, 0.5,0.4,0.3\rangle, \langle b, 0.2,0.3,0.1\rangle, \langle c, 0.8,0.7,0.5\rangle\}$

Here $A_N, B_N, C_N, D_N, E_N, F_N$ are fuzzy neutrosophic semi−open sets but are not fuzzy neutrosophic open sets in (X, τ_N).

4. Fuzzy Neutrosophic Semi−Closed Sets In Fuzzy Neutrosophic Topological Spaces.

In this section, the concept of fuzzy neutrosophic semi−closed set is introduced and also discussed their properties.

Definition 4.1:

Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N). Then A_N is said to be fuzzy neutrosophic semi−closed set in (X, τ_N) if there exists a fuzzy neutrosophic closed (fnC) set such that $fn(fnC)^+ \leq A_N \leq (fnC)$.

Theorem 4.2

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi−closed set if only if $((A_N)^-)^+ \leq A_N$.

Proof:

Sufficiency: Let $((A_N)^-)^+ \leq A_N$. Then for $fnC = (A_N)^-$, we have $(fnC)^+ \leq A_N \leq fnC$. *Necessity:* Let A_N be a fuzzy neutrosophic semi−closed set in (X, τ_N). Then $fn(fnC)^+ \leq A_N \leq fnC$ for some fuzzy neutrosophic semi−closed set fnC. But $fn(A_N)^- \leq fnC$ and thus $((A_N)^-)^+ \leq fn(fnC)^+$ Hence $((A_N)^-)^+ \leq fn(fnC)^+ \leq A_N$.

Theorem 4.3:

Let (X, τ_N) be a fuzzy neutrosophic topological space and A_N be a fuzzy neutrosophic set of (X, τ_N). Then A_N is a fuzzy neutrosophic semi−closed set if and only if complement of $C(A_N)$ is fuzzy neutrosophic semi−open set in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic semi−closed set in (X, τ_N). Then by theorem 4.2, $((A_N)^-)^+ \leq A_N$. Taking complement on both sides, $C(A_N) \leq C(((A_N)^-)^+) = (C(A_N)^-)^-$. By using proposition 1.17, $C(A_N) \leq ((C(A_N))^+)^-$. By Theorem 3.2, $C(A_N)$ is fuzzy neutrosophic semi−open. Conversely let $C(A_N)$ is fuzzy neutrosophic semi−open set. By Theorem 3.2, $C(A_N) \leq ((C(A_N))^+)^-$. Taking complement on both sides, $A_N \geq C\left(\left(C(A_N)\right)^+\right)^- = (C(C(A_N))^+)^+$. By using proposition 1.17(b), $A_N \geq ((A_N)^-)^+$. By theorem 4.2, A_N is a fuzzy neutrosophic semi−closed set.

Theorem 4.4:

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then intersection of two fuzzy neutrosophic semi−closed sets is a fuzzy neutrosophic semi−closed set in fuzzy neutrosophic topological space (X, τ_N).

Proof:

Let A_N and B_N are fuzzy neutrosophic semi−closed sets in (X, τ_N). Then $((A_N)^-)^+ \leq A_N$ and $((B_N)^-)^+ \leq B_N$. Therefore $A_N \wedge B_N \geq ((A_N)^-)^+ \wedge ((B_N)^-)^+ = ((A_N)^- \vee (B_N)^-)^+ \geq ((A_N \vee B_N)^-)^+$[By proposition 1.18] Hence $A_N \wedge B_N$ is fuzzy neutrosophic semi−closed set in (X, τ_N).

Theorem 4.5:

Let $\{A_{N_i}\}_{i \in \triangle}$ be a collection of fuzzy neutrosophic semi−closed sets in a fuzzy neutrosophic topological space (X, τ_N). Then $\wedge_{i \in \triangle} A_{N_i}$ is fuzzy neutrosophic semi−closed set in (X, τ_N).

Proof:

For each $i \in \triangle$, we have a fuzzy neutrosophic semi−closed set fnC_i such that $fn(fnC_i)^+ \leq A_{N_i} \leq fnC_i$. Then $fn(\wedge_{i \in \triangle} fnC_i)^+ \leq \wedge_{i \in \triangle} fn(fnC_i)^+ \leq \wedge_{i \in \triangle} A_{N_i} \leq \wedge_{i \in \triangle} fnC_i$. Hence let $fnC = \wedge_{i \in \triangle} fnC_i$.

Remark 4.6

The union of any two fuzzy neutrosophic semi−closed sets need not be a fuzzy neutrosophic semi−closed set in (X, τ_N) as shown by the following example.

Example 4.7:

Let $X = \{a, b, c\}$ and $A_N = \{\langle a, 0.1, 0.5, 0.5\rangle\}$, $B_N = \{\langle a, 0, 0.9, 0.2\rangle\}$, $C_N = \{\langle a, 0.9, 0.2, 0.1\rangle\}$.

Then $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, B_N \vee C_N, A_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, A_N \wedge C_N, A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topological space on (X, τ_N).

Here $((1 - B_N)^+)^- = 1 = (1 - B_N)^+ = 0$ and $((1 - C_N)^+)^- = 1 = (1 - C_N)^+ = 0$ are fuzzy neutrosophic semi−closed sets in (X, τ_N).

But $((1 - B_N) \vee (1 - C_N)^-)^+ = ((1 - B_N \vee C_N)^-)^+ = (1 - B_N \vee C_N)^+ = A_N \neq 0$ is not a fuzzy neutrosophic semi−closed set in (X, τ_N).

Theorem 4.8:

Let A_N be fuzzy neutrosophic semi−closed set in the fuzzy neutrosophic topological space (X, τ_N) and suppose $(A_N)^+ \leq B_N \leq A_N$. Then B_N is fuzzy neutrosophic semi−closed set in (X, τ_N).

Proof:

There exists a fuzzy neutrosophic semi−closed set (fnC) such that $fn(fnC)^+ \leq A_N \leq fnC$. Then $B_N \leq fnC$. But $fn(fnC)^+ \leq (A_N)^+$and thus $fn(fnC)^+ \leq B_N$. Hence $fn(fnC)^+ \leq B_N \leq fnC$ and B_N is fuzzy neutrosophic semi−closed set in (X, τ_N).

Theorem 4.9:

Every fuzzy neutrosophic closed set in the fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi−closed set in (X, τ_N).

Proof:

Let A_N be fuzzy neutrosophic closed set in fuzzy neutrosophic topological space (X, τ_N). Then $(A_N)^- = A_N$. Also $((A_N)^-)^+ = (A_N)^-$. This implies that $((A_N)^-)^+ \leq A_N$. Hence by theorem 3.2, A_N is a fuzzy neutrosophic semi−closed set in (X, τ_N).

Remark 4.10:

In a fuzzy neutrosophic topological space, every fuzzy neutrosophic semi−closed set in (X, τ_N) is need not to be fuzzy neutrosophic closed set in (X, τ_N). For the following example.

Example 4.11:

Let $X = \{a, b, c\}$ and with $\tau_N = \{0_N, A_N, B_N, C_N, D_N, E_N, 1_N\}$. Some of the fuzzy neutrosophic semi−open sets are

$A_N = \{\langle a, 0.3, 0.6, 0.5\rangle, \langle b, 0.9, 0.7, 0.1\rangle, \langle c, 0.4, 0.6, 0.1\rangle\}$

$B_N = \{\langle a, 0.7, 0.4, 0\rangle, \langle b, 0.9, 0.6, 0.1\rangle, \langle c, 0.8, 0.5, 0.5\rangle\}$

$C_N = \{\langle a, 0.5, 0.4, 0.3\rangle, \langle b, 0.1, 0.3, 0.9\rangle, \langle c, 0.1, 0.4, 0.4\rangle\}$

$D_N = \{\langle a, 0, 0.6, 0.7\rangle, \langle b, 0.1, 0.4, 0.9\rangle, \langle c, 0.5, 0.5, 0.8\rangle\}$

$E_N = \{\langle a, 0.9, 0.4, 0.2\rangle, \langle b, 0.9, 0.2, 0\rangle, \langle c, 0.1, 0.2, 0.3\rangle\}$

Now, $((1 - C_N)^-)^+ = (1 - C_N)^+ = 0$, $((1 - D_N)^-)^+ = (1 - D_N)^+ = 0$,

$((1 - E_N)^-)^+ = (1 - E_N)^+ = 0$.

Here the fuzzy neutrosophic semi−closed sets are $1 - C_N, 1 - D_N, 1 - E_N$ but are not a fuzzy neutrosophic closed sets in (X, τ_N).

5. Fuzzy Neutrosophic Semi−Interior In Fuzzy Neutrosophic Topological Spaces.

In this section, we introduce the fuzzy neutrosophic semi−interior and their properties in fuzzy neutrosophic topological space.

Definition 5.1:

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi−interior of A_N is the union of all fuzzy neutrosophic semi−open sets of (X, τ_N) contained in A_N.

That is, $fnS(A_N)^+ = \vee\{G_N : G_N \text{ is a fuzzy neutrosophic semi} - \text{open set in } (X, \tau_N) \text{ and } G_N \leq A_N\}$

Proposition 5.2

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have

(i) $fnS(A_N)^+ \leq A_N$

(ii) A_N is $fnSO$ set in $(X, \tau_N) \iff fnS(A_N)^+ = A_N$

(iii) $fnS\,(fnS(A_N)^+)^+ = fnS(A_N)^+$

(iv) If $A_N \leq B_N$ then $fnS(A_N)^+ \leq fnS(B_N)^+$.

Proof:

(i) follows from definition 5.1

Let A_N be fuzzy neutrosophic semi−open set in (X, τ_N). Then $A_N \leq fnS(A_N)^+$. Conversely assume that $A_N = fnS(A_N)^+$. By using definition 5.1, A_N is a fuzzy neutrosophic semi−open set in (X, τ_N). Thus (ii) proved.

By using (ii), $fnS\,(fnS(A_N)^+)^+ = fnS(A_N)^+$. This proved (iii).

Since $A_N \leq B_N$ by using (i), $fnS(A_N)^+ \leq A_N \leq B_N$. Thus $fnS(A_N)^+ \leq B_N$. By (iii), $fnS\,(fnS(A_N)^+)^+ = fnS(B_N)^+$. Thus $fnS(A_N)^+ \leq fnS(B_N)^+$. This proves (iv).

Theorem 5.3

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have

(i) $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+$.

(i) $fnS(A_N \vee B_N)^+ \geq fnS(A_N)^+ \vee fnS(B_N)^+$.

Proof:

Since $A_N \wedge B_N \leq A_N$ and $A_N \wedge B_N \leq B_N$. By using proposition 5.2 (iv), $fnS\,(A_N \wedge B_N)^+ \leq fnS(A_N)^+$ and $fnS(A_N \wedge B_N)^+ \leq fnS(B_N)^+$. This implies that $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+ \longrightarrow (1)$. By using proposition 5.2 (i),

$fnS(A_N)^+ \leq A_N$ and $(B_N)^+ \leq B_N$. This implies that $fnS(A_N)^+ \wedge fnS(B_N)^+ \leq A_N \wedge B_N$. Now applying proposition 5.2(iv), $fnS(fnS(A_N)^+ \wedge fnS(B_N)^+)^+ \leq fnS(A_N \wedge B_N)^+ \longrightarrow (2)$. By (1), $fnS(fnS(A_N)^+)^+ \wedge fnS(fnS(B_N)^+)^+ \leq fnS(A_N \wedge B_N)^+$. By proposition 5.2(iii), $fnS(A_N)^+ \wedge fnS(B_N)^+ \leq fnS(A_N \wedge B_N)^+$. From (1) and (2), $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+$. This implies (i).

Since $A_N \leq A_N \vee B_N$ and $B_N \leq A_N \vee B_N$, by using proposition 5.2 (iv), $fnS(A_N)^+ \leq fnS(A_N \vee B_N)^+$ and $fnS(B_N)^+ \leq fnS(A_N \vee B_N)^+$. This implies that $fnS(A_N)^+ \vee fnS(B_N)^+ \leq fnS(A_N \vee B_N)^+$. Hence (ii).

The following example shows that the equality need not hold in theorem 5.3(ii).

6. Fuzzy Neutrosophic Semi−Closure In Fuzzy Neutrosophic Topological Spaces.

In this section, we introduce the concept of fuzzy neutrosophic semi−closure and their properties in fuzzy neutrosophic topological space.

Definition 6.1:

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi−closure of A_N [$fnS\ cl(A_N)$ for short] is the intersection of all fuzzy neutrosophic semi−closed sets of (X, τ_N) contained in A_N.

That is, $fnS(A_N)^- = \wedge\{K_N : K_N \text{ is a } fnSC \text{ set in } (X, \tau_N) \text{ and } K_N \geq A_N\}$.

Proposition 6.2

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a (X, τ_N),

(i) $C(fnS(A_N)^+) \leq fnS(C(A_N))^-$,

(ii) $C(fnS(A_N)^-) \leq fnS(C(A_N))^+$.

Proof:

(i) follows from definition 5.1, $fnS(A_N)^+ = \vee\{G_N : G_N \text{ is a } fnSO \text{ in } (X, \tau_N) \text{ and } G_N \leq A_N\}$. Taking complement on both sides, $C(fnS(A_N)^+) = C(\vee\{G_N : G_N \text{ is a } fnSO \text{ set in } (X, \tau_N) \text{ and } G_N \leq A_N\}) = \wedge\{C(G) : C(G) \text{ is a } fnSC \text{ set in } (X, \tau_N) \text{ and } C(A_N) \leq C(G_N)\}$. Replacing $C(G_N)$ by K_N, we get $C(fnS(A_N)^+) = \wedge\{K_N : K_N \text{ is a } fnSC \text{ set in } (X, \tau_N) \text{ and } K_N \geq C(A_N)\}$. By definition 6.1, $C(fnS(A_N)^+) = fnS(C(A_N))^-$. This proves (i).

By using (i), $C(fnS(C(A_N))^+) = fnS(C(C(A_N)))^- = fnS(A_N)^-$. Taking complement on both sides, we get $fnS(C(A_N))^+ = C(fnS(A_N)^-)$. Hence proved (ii).

Proposition 6.3

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have

(i) $A_N \leq fnS(A_N)^-$

(ii) A_N is $fnSC$ set in $(X, \tau_N) \Leftrightarrow fnS(A_N)^- = A_N$

(iii) $fnS\,(fnS(A_N)^-)^- = fnS(A_N)^-$

(iv) If $A_N \leq B_N$ then $fnS(A_N)^- \leq fnS(B_N)^-$.

Proof:

(i) follows from definition 5.1

Let A_N be fuzzy neutrosophic semi−closed set in (X, τ_N). By using proposition 4.3, $C(A_N)$ be fuzzy neutrosophic semi−open set in (X, τ_N). By proposition 6.2 (ii), $fnS(C(A_N))^+ = C(A_N) \Leftrightarrow C(fnS(A_N)^-) = C(A_N) \Leftrightarrow fnS(A_N)^- = A_N$. Thus (ii) proved.

By using (ii), $fnS\,(fnS(A_N)^-)^- = fnS(A_N)^-$. This proved (iii).

Since $A_N \leq B_N$, $C(A_N) \leq C(B_N)$. By proposition 5.2 (iv), $fnS(C(B_N))^+ \leq fnS(C(A_N))^+$. Taking complement on both sides, $C(fnS(C(B_N))^+) \geq C(fnS(C(A_N))^+)$. By proposition 6.2 (ii), $fnS(A_N)^- \leq fnS(B_N)^-$. This proves (iv).

Proposition 6.4

Let A_N be a fuzzy neutrosophic set in a fuzzy neutrosophic topological space (X, τ_N). Then $fn\,(A_N)^+ \leq fnS\,(A_N)^+ \leq A_N \leq fnS\,(A_N)^- \leq fn\,(A_N)^-$.

Proof:

It follows from the definitions of corresponding operators.

Proposition 6.5

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have

(i) $fnS(A_N \vee B_N)^- = fnS(A_N)^- \vee fnS(B_N)^-$ and

(ii) $fnS(A_N \wedge B_N)^- \leq fnS(A_N)^- \wedge fnS(B_N)^-$.

Proof:

Since $fnS(A_N \vee B_N)^- = fnS(C(C(A_N \vee B_N)))^-$, by using proposition 6.2(i), $fnS(A_N \vee B_N)^- = C(fnS(C(A_N \vee B_N))^+) = C(fnS(C(A_N) \wedge C(B_N))^+)$. Again using proposition 5.3 (i), $fnS(A_N \vee B_N)^- = C(fnS(C(A_N))^+ \wedge fnS(C(B_N))^+) = fnS(C(C(A_N)) \vee fnS(C(C(B_N)))^- = fnS(A_N)^- \vee fnS(B_N)^-$. This proved (i).

Since $A_N \wedge B_N \leq A_N$ and $A_N \wedge B_N \leq B_N$. By using proposition 6.3 (iv), $fnS\,(A_N \wedge B_N)^- \leq fnS(A_N)^-$ and $fnS(A_N \wedge B_N)^- \leq fnS(B_N)^-$. This implies that $fnS(A_N \wedge B_N)^- \leq fnS(A_N)^- \wedge fnS(B_N)^-$. This proves (ii).

Proposition 6.7

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have

(i) $fnS(A_N)^- \geq A_N \vee fnS(\,fnS(A_N)^+)^-$,

(ii) $fnS(A_N)^+ \leq A_N \wedge fnS(\,fnS(A_N)^-)^+$,

$(iii) fn(fnS(A_N)^-)^+ \leq fn(fn(A_N)^-)^+$,
$(iv) fn(fnS(A_N)^-)^+ \geq fn(fnS(fnS(A_N)^+)^-)^+$

Proof:

By using proposition 6.3 (i), $A_N \leq fnS(A_N)^- \longrightarrow (1)$. Again By using proposition 5.2 (i), $fnS(A_N)^+ \leq A_N$. Then $fnS(fnS(A_N)^+)^- \leq fnS(A_N)^- \longrightarrow (2)$. By (1) and (2) we have, $A_N \vee fnS(fnS(A_N)^+)^- \leq fnS(A_N)^-$. This proves (i).

By proposition 5.2 (i), $fnS(A_N)^+ \leq A_N \longrightarrow (1)$. Again by using proposition 6.3 (i), $A_N \leq fnS(A_N)^-$. Then $fnS(A_N)^+ \leq fnS(fnS(A_N)^-)^+ \longrightarrow (2)$. From (1) and (2) we have $fnS(A_N)^+ \leq A_N \wedge fnS(fnS(A_N)^-)^+$. This proves (ii).

By proposition 6.4, $fnS(A_N)^- \leq fn(A_N)^-$. We get $fn(fnS(A_N)^-)^+ \leq fn(fn(A_N)^-)^+$. Hence (iii).

By (i), $fnS(A_N)^- \geq A_N \vee fnS(fnS(A_N)^+)^-$. We have $fn(fnS(A_N)^-)^+ \geq fn(A_N \vee fnS(fnS(A_N)^+)^-)^+$. Since $fn(A_N \vee B_N)^+ \geq fn(A_N)^+ \vee fn(B_N)^+$, $fn(fnS(A_N)^-)^+ \geq fn(A_N)^+ \vee fn(fnS(fnS(A_N)^+)^-)^+ \geq fn(fnS(fnS(A_N)^+)^-)^+$. Hence proves (iv).

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