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APPLICATION OF ROHIT TRANSFORM TO PARABOLIC AND ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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Abstract

We presented a brief description of the basic properties of the integral tramsform, Rohit transform, based on the definition of the transform as defined by Rohit, and we demonstrated with various examples its applications in the solutions of parabolic and elliptic differential equations. Rohit transform is a new transform which has been proposed from Laplace transform. It has been applied to solve ordinary different equations; the equation of motion of a particle, simple harmonic motion problem, electric circuit problem and other equations. In this research, the new integral transform, Rohit transform is used to solve parabolic partial differential equations (Heat equations) and elliptic partial differential equations (wave equations). The boundary conditions are all homogenous boundary conditions.

Keywords: Rohit transform, parabolic differential equation, elliptic differential equation, heat equation, wave equation, integral transform and Laplace transform.

1. Introduction

Heat and wave equations describe so many real life events. Their applications are seen mainly in physics, engineering, mechanics, in fact in all natural sciences. They cut across all bodies. Although heat and wave are not measured directly, they have physical interpretations. Analyzing the temperature distribution in a given system, heat equation is usually used. In forecasting weather, predicting earthquakes or any other natural disaster, partial differential equations such as wave equations are often used.

In the analysis of these differential equations, numerical approach and analytical approach can be applied. In using the analytical approach to finding the solution of any of the two differential equations, many researchers have applied different transform methods. Bahuguna et al [2], applied Laplace transform method to solve one-dimensional heat and wave equations with nonlocal conditions. Patil [9] solved wave equation by double Laplace and double Sumudu transform. for numerical approach, [19] used finite difference scheme with stability conditions to solve heat equation for several thermal diffusivity, [15] analyzed heat conduction equation arising from heat diffusion with a new numerical approach. Geleta et al [6], applied both numerical and analytical approach to heat equation. For the application of Elzaki transform to heat and wave equations, [5], [14] and [20] solved partial differential equations by Elzaki transform method. [1] and [8] applied Kamal transform method to partial differential equations. Chenxi [4] used Fourier transform to solve heat and wave equations. [16] applied residue theory to one-dimensional heat equation. [7], [17] and [18] applied various analytic methods to heat equations. In advance mathematical methods for science and engineering [3], heat and wave equations are solved using various methods.

Rohit [10] proposed a new integral transform, Rohit transform which has been applied in different ways. Rohit ea al [11] studied the response of an undamped forced oscillator using Rohit transform method. Response of RLC network circuit with steady source was investigated by [12] via Rohit transform. Rohit and Diksha applied Rohit transform method to physical and basic sciences problems. This research, followed suit in the application of Rohit transform to parabolic (heat) equations and elliptic (wave) equations.

2. Definition of Rohit Transform

Rohit transform of a continuous function $f(t)$, $t > 0$ is defined by [10] as

$$R\{f(t)\} = r^3 \int_0^\infty e^{-rt} f(t)\,dt = G(r) \qquad r > 0 \tag{1}$$

3. Some basic properties of Rohit transform

(i). Rohit transform of a constant function say k $(k \in \Re)$

$$R\{k\} = r^3 \int_0^\infty e^{-rt} k\,dt = kr^3 \int_0^\infty e^{-rt}\,dt$$

$$R\{k\} = kr^2 \tag{2}$$

(ii) Rohit transform of a variable, say t

$$R\{t\} = r^3 \int_0^\infty e^{-rt} t\,dt$$

Integrating, we get

$$R\{t\} = r \tag{3}$$

(iii) Rohit transform of t^2

$$R\{t^2\} = r^3 \int_0^\infty e^{-rt} t^2\,dt$$

Integrating, we get

$$R\{t^2\} = 2 \tag{4}$$

(iv) If $f(t) = t^3$,

$$R(t^3) = r^3 \int_0^\infty e^{-rt} t^3\,dt = \frac{6}{r} \tag{5}$$

(v)) If $f(t) = t^4$,

$$R(t^4) = r^3 \int_0^\infty e^{-rt} t^4\,dt = \frac{24}{r^2} \tag{6}$$

(vi) Rohit transform of t^n

$$R\{t\} = r^3 \int_0^\infty e^{-rt} t^n\,dt = r^3 \int_0^\infty e^{-rt} t^n\,dt$$

Expanding the right hand side and applying the definition of gamma function as illustrated in [10]

$$R\{t^n\} = \frac{n!}{r^{n-2}} \tag{7}$$

(vii) If $f(t) = e^{at}$,

$$R\left(e^{at}\right)=r^{3}\int_{0}^{\infty}e^{-rt}e^{at}dt=\frac{r^{3}}{r-a} \tag{8}$$

(viii) If $f(t)=\cos at$,

$$R\left(\cos at\right)=r^{3}\int_{0}^{\infty}e^{-rt}\cos at\,dt=\frac{r^{4}}{r^{2}+a^{2}} \tag{9}$$

(ix) If $f(t)=\sin at$,

$$R\left(\sin at\right)=r^{3}\int_{0}^{\infty}e^{-rt}\sin at\,dt=\frac{ar^{3}}{r^{2}+a^{2}} \tag{10}$$

(x) Rohit transform of dependent variable, $v(t)$

$$R\{v(t)\}=r^{3}\int_{0}^{\infty}e^{-rt}v(t)dt$$

We let

$$r^{3}\int_{0}^{\infty}e^{-rt}v(t)\,dt=V(r)$$

Then,

$$R\{v(t)\}=V(r) \tag{11}$$

(xi) Rohit transform of derivatives of $v(t)$

If $f(t)=\frac{dv}{dt}$, then

$$R\left\{\frac{dv}{dt}\right\}=r^{3}\int_{0}^{\infty}e^{-rt}\frac{dv}{dt}dt \tag{12}$$

Integrating the right hand side of equation (12), we get

$$R\left\{\frac{dv}{dt}\right\}=rV(r)-r^{3}v(0) \tag{13}$$

If $f(t)=\frac{d^{2}v}{dt^{2}}$, then

$$R\left\{\frac{d^{2}v}{dt^{2}}\right\}=r^{3}\int_{0}^{\infty}e^{-rt}\frac{d^{2}v}{dt^{2}}dt \tag{14}$$

Integrating the right hand side of equation (14), we get

$$R\left\{\frac{d^{2}v}{dt^{2}}\right\}=r^{2}V(r)-r^{4}v(0)-r^{3}v'(0) \tag{15}$$

If $f(t)=\frac{d^{3}v}{dt^{3}}$, then

$$R\left\{\frac{d^{3}v}{dt^{3}}\right\}=r^{3}\int_{0}^{\infty}e^{-rt}\frac{d^{3}v}{dt^{3}}dt \tag{16}$$

Integrating the right hand side of equation (16), we get

$$R\left\{\frac{d^3v}{dt^3}\right\}=r^3V(r)-r^5v(0)-r^4v'(0)-r^3v''(0) \tag{17}$$

If $f(t)=\dfrac{d^4v}{dt^4}$, then

$$R\left\{\frac{d^4v}{dt^4}\right\}=r^3\int_0^\infty e^{-rt}\frac{d^4v}{dt^4}dt \tag{18}$$

Integrating the right hand side of equation (18), we get

$$R\left\{\frac{d^4v}{dt^4}\right\}=r^4V(r)-r^6v(0)-r^5v'(0)-r^4v''(0)-r^3v'''(0) \tag{19}$$

Then, if $f(t)=\dfrac{d^nv}{dt^n}$

$$R\left\{\frac{d^nv}{dt^n}\right\}=r^3\int_0^\infty e^{-rt}\frac{d^nv}{dt^n}dt$$

(20)

$$R\left\{\frac{d^nv}{dt^n}\right\}=r^nV(r)-r^{n+2}v(0)-r^{n+1}v'(0)-r^nv''(0)-r^{n-1}v'''(0)-\ldots-r^3v^{(n-1)}(0)$$

4. Rohit transform of a function of two variables

Let $f(t)=v(x,t)$, then

$$R\{v(x,t)\}=r^3\int_0^\infty e^{-rt}v(x,t)dt$$

We let $r^3\int_0^\infty e^{-rt}v(x,t)dt=\bar{V}(x,r)$

$$R\{v(x,t)\}=\bar{V}(x,r) \tag{21}$$

5. Rohit transform of partial derivatives of $v(x,t)$

5.1 Rohit transform of first partial derivative of $v(x,t)$ with respect to t

If $f(t)=\dfrac{\partial v}{\partial t}$,

$$R\left\{\frac{\partial v}{\partial t}\right\}=r^3\int_0^\infty e^{-rt}\frac{\partial v}{\partial t}dt \tag{22}$$

Integrating the right hand side of equation (22), we get

$$R\left\{\frac{\partial v}{\partial t}\right\}=-r^3v(x,0)+r\left\{r^3\int_0^\infty e^{-rt}v(x,t)dt\right\}=r\bar{V}(x,r)-r^3v(x,0) \tag{23}$$

5.2 Rohit transform of second partial derivative of $v(x,t)$ with respect to t

If $f(t)=\dfrac{\partial^2 v}{\partial t^2}$,

$$R\left\{\frac{\partial v}{\partial t}\right\}=r^3\int_0^\infty e^{-rt}\frac{\partial^2 v}{\partial t^2}dt \qquad (24)$$

Integrating the right hand side of equation (24), we get

$$R\left\{\frac{\partial^2 v}{\partial t^2}\right\}=-r^4v(x,0)-r^3v'(x.0)+r^2\left\{r^3\int_0^\infty e^{-rt}v(x,t)dt\right\}=r^2\bar{V}(x,r)-r^4v(x,0)-r^3v'(x,0) \qquad (25)$$

5.3 Rohit transform of first partial derivative of $v(x,t)$ with respect to x

If $f(t)=\frac{\partial v}{\partial x}$, then,

$$R\left\{\frac{\partial v}{\partial x}\right\}=r^3\int_0^\infty e^{-rt}\frac{\partial v}{\partial x}dt \qquad (26)$$

Applying Liebnitz theorem of differentiation under integration, we get

$$R\left\{\frac{\partial v}{\partial x}\right\}=r^3\frac{d}{dx}\int_0^\infty e^{-rt}v(x,t)dt \qquad (27)$$

Equation (27) can be written as

$$R\left\{\frac{\partial v}{\partial x}\right\}=\frac{d}{dx}r^3\int_0^\infty e^{-rt}v(x,t)dt=\frac{d\bar{V}(x,r)}{dx} \qquad (28)$$

5.4 Rohit transform of second partial derivative of $v(x,t)$ with respect to x

If $f(t)=\frac{\partial^2 v}{\partial x^2}$,

$$R\left\{\frac{\partial^2 v}{\partial x^2}\right\}=r^3\int_0^\infty e^{-rt}\frac{\partial^2 v}{\partial x^2}dt \qquad (29)$$

Applying Liebnitz theorem of differentiation under integration, we get

$$R\left\{\frac{\partial^2 v}{\partial x^2}\right\}=r^3\frac{d^2}{dx^2}\int_0^\infty e^{-rt}v(x,t)dt \qquad (30)$$

Equation (30) can be written as

$$R\left\{\frac{\partial^2 v}{\partial x^2}\right\}=\frac{d^2}{dx^2}r^3\int_0^\infty e^{-rt}v(x,t)dt=\frac{d^2\bar{V}(x,r)}{dx^2} \qquad (31)$$

6. **Application to Parabolic partial differential equations (Heat conduction problems)**

6.1. **Application I**

$$\frac{\partial^2 v}{\partial x^2}=\frac{1}{2}\frac{\partial v}{\partial t} \qquad 0\le x\le 3,\ t\ge 0 \qquad (32a)$$

$$v(0,t)=v(3,t)=0 \qquad t>0$$
$$v(x,0)=5\sin 4\pi x \qquad 0<x<3 \qquad (32b)$$

Solving equation (32a) using Rohit transform, we find the Rohit transform of equation (32a) and equation (32b).

$$\frac{\partial^2 \bar{V}}{\partial x^2} = \frac{1}{2}\left\{r\bar{V}(x,r) - r^3 v(x,0)\right\} \tag{33a}$$

$$\bar{V}(0,r) = \bar{V}(3,r) = 0$$
$$v(x,0) = 5\sin 4\pi x \tag{33b}$$

Applying the initial condition, equation (33b) on equation (33a), we get

$$\frac{\partial^2 \bar{V}}{\partial x^2} - \frac{1}{2}r\bar{V} = -\frac{5}{2}r^3 \sin 4\pi x \tag{34a}$$

$$\bar{V}(0,r) = \bar{V}(3,r) = 0 \tag{34b}$$

Solving equation (34a), we get

$$\bar{V}(x,r) = Ae^{x\sqrt{\frac{r}{2}}} + Be^{-x\sqrt{\frac{r}{2}}} + \frac{5r^3}{r+32\pi^2}\sin 4\pi x \tag{35}$$

Applying the boundary conditions, equation (34b) on equation (35), we get

$$\bar{V}(x,r) = \frac{5r^3}{r+32\pi^2}\sin 4\pi x \tag{36}$$

We find the inverse Rohit transform of equation (36) and get

$$v(x,t) = 5e^{-32\pi^2 t}\sin 4\pi x \tag{37}$$

6.2. **Application II**

Given the heat conduction problem

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t} \qquad 0 \le x \le 1,\ t \ge 0 \tag{38a}$$

$$v(0,t) = v(1,t) = 1 \qquad t > 0$$
$$v(x,0) = 1 + \sin \pi x \qquad 0 < x < 1 \tag{38b}$$

Solving equation (38a) using Rohit transform, we find the Rohit transform of equation (38a) and equation (38b).

$$\frac{\partial^2 \bar{V}}{\partial x^2} = r\bar{V}(x,r) - r^3 v(x,0) \tag{39a}$$

$$\bar{V}(0,r) = \bar{V}(1,r) = r^2$$
$$v(x,0) = 5\sin 4\pi x \tag{39b}$$

Applying the initial condition, equation (39b) on equation (39a), we get

$$\frac{\partial^2 \bar{V}}{\partial x^2} - r\bar{V} = -r^3(1 + \sin \pi x) \tag{40a}$$

$$\bar{V}(0,r) = \bar{V}(1,r) = r^2 \tag{40b}$$

Solving equation (40a), we get

$$\bar{V}(x,r) = Ae^{x\sqrt{r}} + Be^{-x\sqrt{r}} + r^2 + \frac{r^3}{r+\pi^2}\sin \pi x \tag{41}$$

Applying the boundary conditions, equation (40b) on equation (41), we get

$$\bar{V}(x,r) = r^2 + \frac{r^3}{r+\pi^2}\sin \pi x \tag{42}$$

We find the inverse Rohit transform of equation (42) and get

$$v(x,t) = 1 + e^{-\pi^2 t}\sin \pi x \tag{43}$$

6.3. **Application III**

Given the non-homogeneous heat problem

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} = e^{-t}\sin x \qquad 0 \le x \le \pi,\ t \ge 0 \tag{44a}$$

$$\begin{aligned} &v(0,t) = v(\pi,t) = 0 \qquad t > 0 \\ &v(x,0) = \sin 3x \qquad 0 < x < \pi \end{aligned} \tag{44b}$$

Solving equation (44a) using Rohit transform, we find the Rohit transform of equation (44a) and equation (44b).

$$r\bar{V} - r^3 v(x,0) - \frac{\partial^2 \bar{V}}{\partial x^2} = \frac{r^3}{r+1}\sin x \tag{45a}$$

$$\begin{aligned} &\bar{V}(0,r) = \bar{V}(\pi,r) = 0 \\ &v(x,0) = \sin 3x \end{aligned} \tag{45b}$$

Applying the initial condition in equation (45b) on equation (45a), we get

$$\frac{\partial^2 \bar{V}}{\partial x^2} - r\bar{V} = -r^3\sin 3x - \frac{r^3}{r+1}\sin x \tag{46a}$$

$$\bar{V}(0,r) = \bar{V}(\pi,r) = 0 \tag{46b}$$

Solving equation (46a), we get

$$\bar{V}(x,r) = Ae^{x\sqrt{r}} + Be^{-x\sqrt{r}} + \frac{r^3}{(r+1)^2}\sin x + \frac{r^3}{r+9}\sin 3x \tag{47}$$

Applying the boundary conditions, equation (46b) on equation (47), we get

$$\bar{V}(x,r) = \frac{r^3}{(r+1)^2}\sin x + \frac{r^3}{r+9}\sin 3x \tag{48}$$

We find the inverse Rohit transform of equation (48) and get

$$v(x,t) = \frac{1}{24}e^{-t}t^4\sin x + e^{-9t}\sin 3x \tag{49}$$

7. **Application to Elliptic partial differential equations (Wave equations)**

7.1. **Application IV**

Given the one-dimensional wave equation

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{16}\frac{\partial^2 v}{\partial t^2} \qquad 0 \le x \le 2,\ t \ge 0 \tag{50a}$$

$$\begin{aligned} &v(0,t) = v(2,t) = 0 \qquad && t > 0 \\ &v(x,0) = 6\sin \pi x - 3\sin 4\pi x \qquad && 0 < x < 2 \\ &\frac{\partial v(x,0)}{\partial t} = 0 \qquad && 0 < x < 2 \end{aligned} \tag{50b}$$

Solving equation (50a) using Rohit transform, we find the Rohit transform of equation (50a) and equation (50b).

$$\frac{d^2\bar{V}(x,r)}{dx^2}=\frac{1}{16}\left\{r^2\bar{V}-r^4v(x,0)-r^3v_t(x,0)\right\} \tag{51a}$$

$$\bar{V}(0,r)=\bar{V}(2,r)=0 \tag{51b}$$

Applying the initial conditions in equation (50b) in equation (51a), we get

$$\frac{d^2\bar{V}}{dx^2}-\frac{r^2}{16}\bar{V}=-\frac{r^4}{16}(6\sin\pi x-3\sin 4\pi x) \tag{52}$$

Solving equation (52), we get

$$\bar{V}(x,r)=Ae^{\frac{r}{4}x}+Be^{-\frac{r}{4}x}+\frac{6r^4}{r^2+16\pi^2}\sin\pi x+\frac{3r^4}{r^2+16^2\pi^2}\sin 4\pi x \tag{53}$$

Applying the boundary conditions, equation (51b) in equation (53), we get

$$\bar{V}(x,r)=\frac{6r^4}{r^2+16\pi^2}\sin\pi x+\frac{3r^4}{r^2+16^2\pi^2}\sin 4\pi x \tag{54}$$

Next, we find the inverse Rohit transform of equation (54)

$$v(x,t)=6\cos 4\pi t\sin\pi x-3\cos 16\pi t\sin 4\pi x \tag{55}$$

7.2. **Application V**

Given the wave equation with non-homogenous initial conditions

$$\frac{\partial^2 v}{\partial x^2}=\frac{\partial^2 v}{\partial t^2} \qquad 0\le x\le 1,\ t\ge 0 \tag{56a}$$

$$\begin{aligned} &v(0,t)=v(1,t)=0 && t>0\\ &v(x,0)=\sin\pi x && 0<x<1\\ &\frac{\partial v(x,0)}{\partial t}=-\sin\pi x && 0<x<1 \end{aligned} \tag{56b}$$

Solving equation (56a) using Rohit transform, we find the Rohit transform of equation (56a) and equation (56b).

$$\frac{d^2\bar{V}(x,r)}{dx^2}=\left\{r^2\bar{V}-r^4v(x,0)-r^3v_t(x,0)\right\} \tag{57a}$$

$$\bar{V}(0,r)=\bar{V}(1,r)=0 \tag{57b}$$

Applying the initial conditions in equation (56b) in equation (57a), we get

$$\frac{d^2\bar{V}}{dx^2}-r^2\bar{V}=-(r^4-r^3)\sin\pi x \tag{58}$$

Solving equation (58), we get

$$\bar{V}(x,r)=Ae^{\sqrt{r}x}+Be^{-\sqrt{r}x}+\frac{r^4-r^3}{r^2+\pi^2}\sin\pi x \tag{59}$$

Applying the boundary conditions, equation (57b) in equation (59), we get

$$\bar{V}(x,r)=\frac{r^4}{r^2+\pi^2}\sin\pi x-\frac{r^3}{r^2+\pi^2}\sin\pi x \tag{60}$$

Next, we find the inverse Rohit transform of equation (60)

$$v(x,t)=\cos\pi t\sin\pi x-\frac{1}{\pi}\sin\pi t\sin\pi x \tag{61}$$

7.3. **Application VI**

Given, a one-dimensional non-homogenous wave equation

$$\frac{\partial^2 v}{\partial x^2}+\frac{1}{c^2}\frac{\partial^2 v}{\partial t^2}+k\sin\pi x=0 \qquad 0\le x\le l,\ t\ge 0 \tag{62a}$$

$$\begin{aligned} v(0,t)&=v(l,t)=0 && t>0\\ v(x,0)&=\frac{\partial v(x,0)}{\partial t}=0 && 0<x<l \end{aligned} \tag{62b}$$

Solving equation (62a) using Rohit transform, we find the Rohit transform of equation (62a) and equation (62b).

$$\frac{d^2\bar{V}(x,r)}{dx^2}-\frac{1}{c^2}\left\{r^2\bar{V}-r^4v(x,0)-r^3v_t(x,0)\right\}+kr^2\sin x=0 \tag{63a}$$

$$\bar{V}(0,r)=\bar{V}(l,r)=0 \tag{63b}$$

Applying the initial conditions in equation (62b) in equation (63a), we get

$$\frac{d^2\bar{V}}{dx^2}-\frac{r^2}{c^2}\bar{V}=-kr^2\sin\pi x \tag{64}$$

Solving equation (64), we get

$$\bar{V}(x,r)=Ae^{\frac{r}{c}x}+Be^{-\frac{r}{c}x}+\frac{kc^2r^2}{r^2+c^2\pi^2}\sin\pi x \tag{65}$$

Applying the boundary conditions, equation (63b) in equation (65), we get

$$\bar{V}(x,r)=\frac{kc^2r^2}{r^2+c^2\pi^2}\sin\pi x \tag{66}$$

Equation (66) can be rewritten in the form

$$\bar{V}(x,r)=\frac{kc^2r^4}{r^2\left(r^2+c^2\pi^2\right)}\sin\pi x \tag{67}$$

Next, we decompose the right hand side of equation (67) into sum of its partial fractions and we get

$$\bar{V}(x,r)=\left(\frac{k}{\pi^2r^2}-\frac{k}{\pi^2\left(r^2+c^2\pi^2\right)}\right)r^4\sin\pi x \tag{68}$$

We find the inverse Rohit transform of equation (68)

$$v(x,t)=\frac{k}{\pi^2}\sin\pi x-\frac{k}{\pi^2}\cos c\pi t\sin\pi x \tag{69}$$

8. Conclusion

This research presented the application of Rohit transform to heat and wave equations. Rohit transform of partial derivatives of a given function with respect to the two independent variables respectively were obtained. Rohit transform was able to transform the partial differential equations in two variables to ordinary differential equations in one variable which were thereafter solved. Rohit transform removed the partial derivatives with respect to time as in Laplace. The results obtained satisfied both the boundary and initial conditions. The technique was found to be less computational and efficient.

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