IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

ON πSC^*-CLOSED SET IN TOPOLOGICAL SPACES

NEERAJ KUMAR TOMAR AND M. C. SHARMA

DEPARTMENT OF APPLIED MATHEMATICS, GAUTAM BUDDHA UNIVERSITY, GREATER NOIDA, UTTAR PRADESH 201312, INDIA

DEPARTMENT OF MATHEMATICS, N. R. E. C. COLLEGE, KHURJA, UTTAR PRADESH 203131, INDIA

ABSTRACT. In this paper we introduced a new class of closed sets in a topological space called, πSC^*-closed sets and some of its characteristics are investigated. Further we studied the concepts of πSC^*-open sets and πSC^*-$T_{1/2}$ space.

INTRODUCTION

Levine [7] and Andrijevic [2] introduced the concept of generalized open sets and *b*-open sets respectively in topological spaces. The class of *b*-open sets is contained in the class of semipre-open sets and contains the class of semi-open and the class of pre-open sets. Since then, several researches were done and the notion of generalized semi-closed, generalized pre-closed and generalized semipre- open sets were investigated in [3, 8, 5]. In 1968 Zaitsev [13] defined $\bar{\pi}$-closed sets. Later Dontchevand Noiri [6] introduced the notion of πg-closed sets. Park [10] defined πgp-closed sets. Then Aslim, Caksu and Noiri [4] introduced the notion of πgs-closed sets. The idea of πgb-closed sets were introduced by D.Sreeja and S.Janaki [12]. A.Chandrakala and K.Bala Deepa Arasi[14] define SC^*-closed sets. Later the properties and characteristics of πgb-closed sets were introduced by Sinem Caglar and Gulhan Ashim [1]. The aim of this paper is to investigate the notion of πSC^*- closed sets and its properties. In section 2, we study the basic properties of πSC^*-closed sets. In section 3, some characteristics of πSC^*-closed sets are introduced and the idea of πSC^*-$T_{1/2}$ space is discussed.

1. PRELIMINARIES AND NOTATIONS

In what follows, spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated and $f : (X, \tau) \to (Y, \sigma)$ (or simply $f : X \to Y$) denotes a function f of a space (X, τ) into a space (Y, σ). Let A be a subset of a space X. The closure and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively.

1.1. **Definition:** A subset A of a space X is said to be
(1) a semi-closed set if $int(cl(A)) \subseteq A$.
(2) a α-closed set if $cl(int(cl(A))) \subseteq A$.
(3) a pre-closed set if $cl(int(A)) \subseteq A$.
(4) a semipre-closed set if $int(cl(int(A))) \subseteq A$.
(5) a regular-closed set if $A = cl(int(A))$.
(6) a b-closed set if $cl(int(A)) \cap int(cl(A)) \subseteq A$.
(7) a b^*-closed set if $int(cl(A)) \subset U$, whenever $A \subset U$ and U is b-open.
(8) a SC^*-closed set if $scl(A) \subset U$, whenever $A \subset U$ and U is c^*-open.

Date: August 23, 2024.
2020 *Mathematics Subject Classification.* 54A05, 54C08, 54C10, 54D15.
Key words and phrases. πSC^*-closed, πSC^*-$T_{1/2}$ - space, πSC^*-open, πSC^*-closure operator.

The complements of the above-mentioned sets are called semi open, α-open, pre-open, semipre- open, regular open, b-open, $b*$-open, and $SC*$-open sets respectively.

The intersection of all semi closed (resp. α-closed, preclosed, semipre-closed, regular closed, b-closed and $SC*$-closed) subsets of (X, τ) containing A is called the semi closure (resp. α-closure, pre-closure, semipre-closure, regular closure, b-closure and $SC*$-closure) of A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$, $pcl(A)$, $spcl(A)$, $rcl(A)$, $bcl(A)$ and $SC*\text{-}cl(A)$). A subset A of (X, τ) is called clopen if it is both open and closed in (X, τ).

1.2. **Definition:** A subset A of a space (X, τ) is called π-closed [13] if A is a finite intersection of regular closed sets.

1.3. **Definition:** A subset A of a Space (X, τ) is called $c*$-open [9] if $int(cl(A)) \subset A \subset cl(int(A))$.

1.4. **Definition:** A subset A of a space (X, τ) is called w-closed [11] (weakly closed) if $cl(A) \subseteq U$, whenever $A \subset U$ and U is semi-open in (X, τ).

1.5. **Definition:** A subset A of a space (X, τ) is called

(1) a g-closed set if $cl(A) \subset U$, whenever $A \subset U$ and U is open in (X, τ).
(2) a gp-closed set if $pcl(A) \subset U$, whenever $A \subset U$ and U is open in (X, τ).
(3) a gs-closed set if $scl(A) \subset U$, whenever $A \subset U$ and U is open in (X, τ).
(4) a gb-closed set if $bcl(A) \subset U$, whenever $A \subset U$ and U is open in (X, τ).
(5) a $g\alpha$-closed set if $\alpha cl(A) \subset U$, whenever $A \subset U$ and U is open in (X, τ).
(6) a πg-closed set if $cl(A) \subset U$, whenever $A \subset U$ and U is π-open in (X, τ).
(7) a $\pi g\alpha$-closed set if $\alpha cl(A) \subset U$, whenever $A \subset U$ and U is π-open in (X, τ).
(8) a πgp-closed set if $pcl(A) \subset U$, whenever $A \subset U$ and U is π-open in (X, τ).
(9) a πgs-closed set if $scl(A) \subset U$, whenever $A \subset U$ and U is π-open in (X, τ).
(10) a πgb-closed set if $bcl(A) \subset U$, whenever $A \subset U$ and U is π-open in (X, τ).

Complement of π-closed set and w-closed set is called π-open set and w-open set.
Complement of $c*$-open set is called $c*$-closed set.
Complement of g-closed, gp-closed, gs-closed, gb-closed, $g\alpha$-closed, πg-closed, $\pi g\alpha$-closed, πgp-closed, πgs-closed, and πgb-closed sets are called g-open, gp-open, gs-open, gb-open, $g\alpha$-open, πg-open, $\pi g\alpha$-open, πgp-open, πgs-open, and πgb-open sets respectively.

1.6. **Definition:** Let (X, τ) be a topological space then a set $A \subseteq (X, \tau)$ is said to be Q-set if $int(cl(A)) = cl(int(A))$.

2. $\pi SC*$-Closed Sets in Topological Spaces

2.1. **Definition:** A subset A of a space (X, τ) is said if $\pi SC*$-closed $SC*\text{-}cl(A) \subseteq U$, whenever $A \subset U$ and U is π-open in (X, τ).

2.2. **Example.** Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$.Then the $\pi SC*$-closedsets are $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$.

2.3. **Definition:** A subset A of a space (X, τ) is said if $\pi SC*$-open set if its complement is $\pi SC*$-closed.

2.4. **Example.** Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then the $\pi SC*$-open sets are $\{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

2.5. **Theorem.** Let X be topological spaces. Then every w-closed set is $\pi SC*$-closed.
Proof. Let A be a w-closed set. Let U be a π-open set containing A. Since every π-open setis open, U is open. Since A is w-closed, $cl(A) \subset U$. Since $SC*\text{-}cl(A) \subset cl(A)$, $SC*\text{-}cl(A) \subset U$. Therefore, A is $\pi SC*$-closed.

2.5.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.5.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Here $A = \{a, b\}$ is $\pi SC*$-closed but it is not w-closed.

2.6. **Theorem.** Let X be topological spaces. Then every closed set is $\pi SC*$-closed.
Proof. Let A be a closed set. Since every closed set is w-closed and by **Theorem 2.5**, A is $\pi SC*$-closed.

2.6.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.6.2. *Example.* Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Here $A =$

$\{a, b\}$ is πSC^*-closed but it is not closed.

2.7. **Theorem.** Let X be topological spaces. Then regular closed set is πSC^*-closed.
Proof. Let A be a regular closed set. Since every regular closed set is closed and by **Theorem2.6**, A is πSC^*-closed.

2.7.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.7.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Here $A = \{a, b\}$ is πSC^*-closed but it is not regular-closed.

2.8. **Theorem.** Let X be topological spaces. Then π-closed set is πSC^*-closed.
Proof. Let A be a π-closed set. Since every π-closed set is closed and by **Theorem 2.6**, A is πSC^*-closed.

2.8.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.8.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Here $A = \{a\}$ is πSC^*-closed but it is not π-closed.

2.9. **Theorem.** Let X be topological spaces. Then every g-closed set is πSC^* -closed.
Proof. Let A be a g-closed set. Then $cl(A) \subseteq U$ whenever, $A \subseteq U$ and U is open. Let U be a π-open set containing A. Since every π-open set is open, U is open. Since A is g -closed, $cl(A) \subseteq U$. Since $SC^*\text{-}cl(A) \subseteq cl(A)$, $SC^*\text{-}cl(A) \subseteq cl(U)$. Therefore, A is πSC^*-closed.

2.9.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.9.2. *Example.* Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Here $A = \{a, b, d\}$ is πSC^*-closed but it is not g-closed.

2.10. **Theorem.** Let X be topological spaces. Then every gs -closed set is πSC^*-closed.
Proof. Let A be a gs-closed set. Let U be a π-open set containing A. Since every π-open set is semi-open, U is semi-open. Since A is gs -closed, $scl(A) \subseteq U$. Therefore, A is πSC^*-closed.

2.10.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.10.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Here $A = \{a, c\}$ is πSC^*-closed but it is not gs-closed.

2.11. **Theorem.** Every gp-closed set is πSC^*-closed.
Proof. Let A be a gp-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Since every π-open set is open, $pcl(A) \subset U$, as $scl(A) \subset pcl(A) \subset U$,$SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence $A$is πSC^*-closed.

2.11.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.11.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a, b\}, X\}$. Here $A = \{a, b\}$ is πSC^*-closed but itis not gp-closed.

2.12. **Theorem.** Every $g\alpha$-closed set is πSC^*-closed.
Proof. Let A be a $g\alpha$-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Sinceevery π-open set is open, $\alpha cl(A) \subset U$, as $scl(A) \subset \alpha cl(A) \subset U$, $SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence A is πSC^*-closed.

2.12.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.12.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, X\}$. Here $A = \{a\}$ is πSC^*-closed but it isnot $g\alpha$-closed.

2.13. **Theorem.** Every gb-closed set is πSC^*-closed.
Proof. Let A be a gb-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Sinceevery π-open set is open, $bcl(A) \subset U$, as $scl(A) \subset bcl(A) \subset U$, $SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence $A$is πSC^*-closed.

2.13.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.13.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, c\}, X\}$. Here $A = \{a, c\}$ is πSC^*-closedbut it is not gb-closed.

2.14. **Theorem.** Every b^*-closed set is πSC^*-closed.
Proof. Let A be a b^*-closed subset of (X, τ) such that $A \quad U$ and U is π-open in X. Sinceevery π-open set is b-open, and A is b^*-closed, as $scl(A) \subset int(bcl(A)) \subset U$, $SC^*\text{-}cl(A) \subset U$. Hence A is πSC^*-closed.

2.14.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.14.2. *Example.* Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Here $A = \{a\}$ is πSC^*-closed but it is not b^*-closed.

2.15. **Theorem.** Every πg-closed set is πSC^*-closed.
Proof. Let A be a πg-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Then $cl(A) \subset U$, as $SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence A is πSC^*-closed.

2.15.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.15.2. *Example.* Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Here $A = \{c\}$ is πSC^*-closed but it is not πg -closed.

2.16. **Theorem.** Every $\pi g\alpha$-closed set is πSC^*-closed.
Proof. Let A be a $\pi g\alpha$-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Then $\alpha cl(A) \subset U$, and as $scl(A) \subset \alpha cl(A) \subset U$, $SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence A is πSC^*-closed.

2.16.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.16.2. *Example.* Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Here $A = \{a\}$ is πSC^*-closed but it is not $\pi g\alpha$-closed.

2.17. **Theorem.** Every πgp-closed set is πSC^*-closed.
Proof. Let A be a πgp-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Then $pcl(A) \subset U$, and as $scl(A) \subset pcl(A) \subset U$, $SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence A is πSC^*-closed.

2.17.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.17.2. Example. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Here $A = \{a, b\}$ is πSC^*-closed but it is not πgp-closed.

2.18. **Theorem.** Every πgs-closed set is πSC^*-closed.
Proof. Let A be a πgs-closed subset of (X, τ) such that $A \subset U$ and U is π-open in X. Then $scl(A) \subset U$, and as $bcl(A) \subset scl(A) \subset U$, $SC^*\text{-}cl(A) \subset bcl(A) \subset scl(A) \subset U$, $SC^*\text{-}cl(A) \subset scl(A) \subset U$. Hence A is πSC^*-closed.

2.18.1. *Remark.* The converse of the above theorem is not true as seen from the following example.

2.18.2. *Example.* Let X be the real numbers with the usual topology and A be the set of irrationalnumbers in the interval $(0, 2)$. Then A is πSC^*-closed but it is not πgs-closed.

3. Characteristics Of πSC^*-Closed Sets

3.1. **Theorem.** Let X be a topological space. If ϕ and X are the only π-open sets, then all thesubsets of X are πSC^*-closed.
Proof. Let A be a subset of X. If $A = \phi$, then A is πSC^*-closed. If $A \neq \phi$, then X is the only π- open set containing A. This implies, $SC^*\text{-}cl(A) \subset X$. Hence A is πSC^*-closed.

3.2. **Theorem.** Let X be a topological space. If A is πSC^*-closed subset of X such that $A \subseteq B \subseteq SC^*\text{-cl}(A)$ Then B is a πSC^*-closed set in X.
Proof. Let H be a π-open set containing B. Then $A \subseteq H$. Since A is a πSC^*-closed we have $SC^*\text{-}cl(A) \subseteq H$. Since $B \subseteq SC^*\text{-cl}(A)$, we have $SC^*\text{-cl}(B) \subseteq H$. Therefore, B is a πSC^*-closed set in X.

3.3. **Theorem.** Let X be a topological space and A be a subset of X. If A is regular open and πSC^*-closed, then A is both semi-open and semi-closed.
Proof. Assume that A is regular open and πSC^*-closed. Since every regular open set is π-open, we have $SC^*\text{-}cl(A) \subseteq A$. Then $A = SC^*\text{-}cl(A)$. This implies, A is semi-closed. Since A is regular open, we have A is semi-open. Hence A is both semi-open and semi-closed set in X.

3.4. Theorem. Let (X, τ) be a topological space if $A \subset X$ is nowhere dense then A is πSC^*-closed.
Proof. Let $A \subseteq U$ where U is π-open in X. Since A is nowhere dense, $SC^*\text{-}cl(A) = \phi$. Now $SC^*\text{-}scl(A) \subset SC^*\text{-}cl(A) = \phi \subset U$. Therefore A is πSC^*-closed in X.

3.5. Theorem. If $cl(SC^*\text{-}cl(A)) \subset B \subset A$ and A is πSC^*-open, then B is πSC^*-open.
Proof. Let F be a π-closed set such that $F \subseteq B$. since $B \subseteq A$ we get $F \subseteq A$. Given A is πSC^*-open thus $F \subset cl(SC^*\text{-}cl(A)) \subset cl(SC^*\text{-}cl(B))$. Therefore B is πSC^*-open.

3.6. Definition. A space (X, τ) is called a $\pi SC^*\text{-}T_{1/2}$ space if every πSC^*-closed set is π-closed.
3.7. Theorem. For a topological space (X, τ) the following are equivalent.
1. X is $\pi SC^*\text{-}T_{1/2}$
2. for all $A \subset X$, A is πSC^*-open if A is π-open.
Proof. (1) ⇒(2) Let $A \subseteq X$ be πSC^*-open. Then $(X - A)$ is πSC^*-closed and by (1) $(X - A)$ is π- closed ⇒A is π-open. Conversely assume A is π-open. Then $(X - A)$ is π-closed. As every π-closed set is πSC^*-closed, $(X - A)$ is πSC^*-closed ⇒A is πSC^*-open.
(2) ⇒(1) Let A be a πSC^*-closed set in X. Then $(X - A)$ is πSC^*-open. Hence by (2) $(X - A)$ π-open ⇒ A is π-closed. Hence X is $\pi SC^*\text{-}T_{1/2}$.

3.8 Theorem. Let (X, τ) be a $\pi SC^*\text{-}T_{1/2}$ space then every singleton set is either π-closed or b^*-open.
Proof. Let $x \in X$ suppose {x} is not π-closed. Then X – {x} is not π-open. Hence X – {x} is trivially πSC^*-closed. Since X is $\pi SC^*\text{-}T_{1/2}$ space, X – {x} is b^*- closed ⇒ {x} is b^*-open.

4. Conclusion

In this paper we have introduced πSC^*- closed sets in topological spaces and studied some ofits basic properties. Also, we have studies the relationship between πSC^*- closed sets with some generalized sets in topological spaces.

References

1. Sinem Caglar Akgun and G Ashm, *On πgb-closed sets and related topics*, International Journal of Mathematical Archive **3** (2012), no. 5, 1873–1884.
2. Dimitrije Andrijević, *On b-open sets*, Matematički Vesnik (1996), no. 205, 59–64.
3. SP Arya, *Characterizations of s-normal spaces*, Indian J. Pure Appl. Math. **21** (1990), no. 8, 717–719.
4. Gulhan Aslim, Aysegul Caksu Guler, and Takashi Noiri, *On π gs-closed sets in topological spaces*, Acta Mathematica Hungarica **112** (2006), no. 4, 275–283.
5. J Dontchev, *On generalizing semi-preopen sets*, Mem. Fac. Sci. Kochi Univ. Ser. A, Math. **16** (1995), 35–48.
6. J Dontchev and T Noiri, *Quasi-normal spaces and π g-closed sets*, Acta Mathematica Hungarica **89** (2000),no. 3, 211–219.
7. Norman Levine, *Generalized closed sets in topology*, Rendiconti del Circolo Matematico di Palermo **19** (1970), 89–96.
8. Haruo MAKI, *Every topological space is pre-t ¡ 1/2*, Mem. Fac. Sci. Kochi Univ. Ser. A Math. **17** (1996), 33–42.
9. S Malathi and S Nithyanantha Jothi, *On pre-generalized c*-closed sets in topological spaces*, Journal of Computer and Mathematical Sciences **8** (2017), no. 12, 720–726.
10. JH Park, MJ Son, and BY Lee, *On πgp-closed sets in topological spaces*, Indian J. Pure Appl. Math., Acta Mathematica Hungarica **112** (2006), no. 4, 257–283.
11. A Pushpalatha, *Studies on generalizations of mappings in topological spaces*, (2000).
12. D Sreeja and C Janaki, *On πgb-closed sets in topological spaces*, International Journal of Mathematical Archive **2** (2011), no. 8, 1314–1320.
13. V Zaitsev, *On certain classes of topological spaces and their bicompactifications*, Dokl. Akad. Nauk SSSR, vol. 178, 1968, pp. 778–779.
14. A. Chandrakala and K. Bala Deepa Arasi, On SC*- closed sets in topological spaces, Science, Technology and Development, Vol.XIV. May 2022, 416-421.