IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

QUEUEING THEORY

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Abstract- This paper characterizes the building pieces of and determines essential queueing frameworks. It begins with a review of a few likelihoods hypothesis and then identifies forms utilized to dissect queueing frameworks, in particular the birth-death preparation. A few straightforward lines are dissected in terms of steady-state induction some time recently the paper examines a few endeavoured fields inquire about on the topic.

1. Introduction

Queueing hypothesis is a department of arithmetic that thinks about and models the act of holding up in lines. This paper will take a brief see into the detailing of queueing hypothesis along with cases of the models and applications of their utilize. The purpose of the paper is to provide the peruser with sufficient expertise in establishing a comprehensive queueing framework into one of the categories we will see at, when possible. Moreover, the peruser ought to start to get it the fundamental thoughts of how to decide valuable data such as normal holding up times from a specific queueing system.

The to begin with paper on queueing hypothesis, “The Hypothesis of Probabilities and Phone Conversations” was distributed in 1909 by A.K. Erlang, presently considered the father of the field. His work with the Copenhagen Phone Company is what incited his introductory attack into the field. He considered the issue of deciding how numerous phone circuits were essential to give phone benefit that would anticipate clients from holding up as well long for an accessible circuit. In creating a arrangement to this issue, he started to realize that the issue of minimizing holding up time was pertinent to numerous areas, and started creating the hypothesis further.

Erlang’s switchboard issue laid the way for cutting edge queueing hypothesis. The chapters on queueing hypothesis and its applications in the book “Operations Inquire about: Applications and Algorithms” by Wayne L. Winston outlines numerous developments of queueing hypothesis and is the book from which the larger part of the investigate of this paper has been done.

In the moment of this paper, we will begin analysing the fundamental queueing demonstration. We will start by looking into the vital probabilistic foundation required to get it the hypothesis. The we will move on to examining documentation, queueing disciplines, birth-death forms, steady-state probabilities, and Little’s queueing equation. In the following segment we will start looking at specific queueing models. We will consider the populace measure, the client capacity, the number of servers, self-service lines, and the machine repair demonstrate, to title a few. We will be calculating steady-state probabilities and holding up times for the models when conceivable, whereas too looking at illustrations and applications. We will conclude the paper by taking a look at a few fields investigate considering the queueing framework at a bank.

2. The Fundamental Queueing Model

To start understanding lines, we must begin with have a few information of likelihood hypothesis. In specific, we will survey the exponential and Poisson likelihood distributions.

2.1. Exponential and Poisson Likelihood Disseminations. The exponential conveyance with parameter λ is given by λe-λt for t ≥ 0. If T is a irregular variable that speaks to interarrival times with the exponential dissemination, at that point P (T ≤ t) = 1 - e-λt and P (T > t) = e-λt.

This dispersion loans itself well to demonstrating client interarrival times or benefit times for a few reasons. The to begin with is the reality that the exponential work is a entirely diminishing work of t. This implies that after an entry has happened, the sum of holding up time until the following entry is more likely to be little than huge. Another vital property of the exponential dispersion is what is known as the no-memory property. The no-memory property recommends that the time until the following entry will never depend on how much time has as of now passed. This makes instinctive sense for a show where we're measuring client entries since the customers' activities are clearly autonomous of one another.

It's too valuable to note the exponential distribution's connection to the Poisson dispersion. The Poisson dispersion is utilized to decide the likelihood of a certain number of entries happening in a given time period. The Poisson conveyance with parameter λ is given by

(λt)ne-λt n!

where n is the number of entries. We discover that if we set n = 0, the Poisson

distribution gives us

e-λt

which is break even with to P (T > t) from the exponential distribution.

The connection here moreover makes sense. After all, we ought to be able to relate the likelihood that zero entries will happen in a given period of time with the likelihood that an interarrival time will be of a certain length. The interarrival time here, of course, is the time between client entries, and in this way is a period of time with zero arrivals.

With these conveyances in intellect, we can start characterizing the input and yield forms of a fundamental queueing framework, from which we can begin creating the demonstrate further.

2.2. The Input Prepare. To start displaying the input prepare, we characterize ti as the time when the ith client arrives. For all i ≥ 1, we characterize Ti = ti+1 -ti to be the ith interarrival time. We too accept that all Ti's are autonomous, ceaseless irregular factors, which we speak to by the arbitrary variable A with likelihood thickness a(t). Ordinarily, A is chosen to have an exponential likelihood dissemination with parameter λ characterized as the entry rate, that is to say, a(t) = λe-λt.

It is simple to appear [W 1045] that if A has an exponential conveyance, at that point for all nonnegative values of t and h,

P (A > t + h|A ≥ t) = P (A > h)

This is an vital result since it reflects the no-memory property of the exponential conveyance, which is an imperative property to take note of if we're displaying interarrival times.

Another dissemination the can be utilized to demonstrate interarrival times (in the event that the exponential conveyance does not appear to be fitting) is the Erlang conveyance. An Erlang conveyance is a nonstop irregular variable whose thickness work relies

on a rate parameter R and a shape parameter k. The Erlang probability density function is

f(t) = R(Rt)k-1e-Rt (k - 1)!

2.3. The Yield Prepare. Much like the input handle, we begin investigation of the yield prepare by expecting that benefit times of diverse clients are autonomous irregular factors spoken to by the arbitrary variable S with likelihood thickness $s(t) = \mu e^{-\mu t}$. We moreover characterize μ as the benefit rate, with units of clients per hour. In a perfect world, the yield prepare can too be demonstrated as an exponential arbitrary variable, as it makes calculation much less complex. Envision an case where four clients are at a bank with three tellers with exponentially dispersed benefit times. Three of them get benefit promptly, whereas the fourth has to hold up for one position to clear. What is the likelihood that the fourth client will be the last one to total service?

Due to the no-memory property of the exponential dispersion, when the fourth client at last steps up to a teller, all three remaining clients have an rise to chance of wrapping up their benefit final, as the benefit time in this circumstance is not represented by how long they have as of now been served. In this way, the reply to the address is 1/3. Shockingly, the exponential dispersion does not continuously speak to benefit times precisely. For a benefit that requires numerous diverse stages of benefit (for illustration, filtering basic supplies, paying for goods, and stowing the foodstuffs), an Erlang conveyance can be utilized with the parameter k rise to to the number of distinctive stages of service.

2.4. Birth-Death Forms. We characterize the number of individuals found in a queueing framework, either holding up in line or in benefit, to be the state of the framework at time t. At t = 0, the state of the framework is going to be rise to to the number of individuals at first in the framework. The beginning state of the framework is critical since it clearly influences the state at a few future t. Knowing this, we can characterize $P_{ij}(t)$ as the likelihood that the state at time t will be j, given that the state at t = 0 was i. For a huge t, $P_{ij}(t)$ will really ended up free of i and approach a constrain π_j. [W 1053] This constrain is known as the steady-state of state j. By and large, if one is looking at the steady-state likelihood of j, it is fantastically troublesome to determine

Figure 1. In a single-server birth-death handle, births include one to the current state and happen at rate λ. Passings subtract one from the current state and happen at rate μ.

the steps of entries and administrations that driven up to the consistent state. Moreover, beginning from a little t, it is moreover exceptionally troublesome to decide when precisely a framework will reach its consistent state, if it exists. In this way, for simplicity's purpose, when we consider a queueing framework, we start by expecting that the steady state has as of now been come to. A birth-death handle is a handle wherein the system's state at any t is a nonnegative numbers. The variable λ_j is known as the birth rate at state j and symbolizes the likelihood of an entry happening over a period of time. The variable μ_j is known as the passing rate at state j and symbolizes the likelihood that a completion of benefit happens over a period of time. In this way, births and passings are synonymous with entries and benefit completions separately. A birth increments the state by one whereas a passing diminishes the state by one. We note that $\mu_0 = 0$, since it must not be conceivable to enter a negative state. Too, in arrange to formally be considered a birth-death prepare, birth and passings must be autonomous of each other. A basic birth-death prepare is outlined in Figure 1.

The likelihood that a birth will happen between t and t + ∆t is $\lambda_j \Delta t$, and such a birth will increment the state from j to j + 1. The likelihood that a passing will happen between t and t + ∆t is $\mu_j \Delta t$, and such a birth will diminish the state from j to j - 1.

2.5. Steady-state Probabilities. In arrange to decide the steady-state likelihood π_j, we have to discover a connection between $P_{ij}(t + \Delta t)$ and $P_{ij}(t)$ for a sensibly measured t. We start by categorizing the potential states at time t from which a framework might conclusion up at state j at time t + ∆t. In arrange to accomplish this, the state at time t must be j, j - 1, j + 1, or a few other esteems. At that point, to calculate π_j, all we have to do is include up the probabilities of the framework finishing at state j for each of these starting categories.

To reach state j from state j - 1, we require one birth to happen between t and ∆t. To reach j from j + 1, we require one passing. To stay at j, we require no births or passings to happen. To reach j from any other state we will require different births or passings. Since we will be in the long run letting ∆t approach zero, we discover that it is outlandish to reach state j from these other states since births and passings are autonomous of each other and won't happen at the same time. Consequently, we as it were required to entirety the probabilities of these to begin with three circumstances happening. That will grant us.

$P_{ij}(t + \Delta t) = [P_{i,j-1}(t)(\lambda_{j-1}\Delta t)] + [P_{i,j+1}(t)(\mu_{j+1}\Delta t)] + [P_{ij}(t)(1 - \mu_j\Delta t - \lambda_j\Delta t)]$

which can be modified as

$P'_{ij}(t) = \lim$

$\Delta t \to \infty$

$P_{ij}(t + \Delta t) - P_{ij}\ \Delta t = \lambda_{j-1}P_{i,j-1}(t)+\mu_{j+1}P_{i,j+1}(t)-P_{ij}(t)\mu_j-P_{ij}(t)\lambda_j$

Since we're attempting to calculate steady-state probabilities, it is fitting to permit t to approach interminability, at which point $P_{ij}(t)$ can be thought of as a steady. At that point $P'_{ij}(t) = 0$ Characterizing the steady-state likelihood $\pi_j = \text{limit}{\to\infty}\ P_{ij}(t)$, we can substitute further.

$\lambda_{j-1}\pi_{j-1} + \mu_{j+1}\pi_{j+1} - \pi_j\mu_j - \pi_j\lambda_j = 0$

$\lambda_{j-1}\pi_{j-1} + \mu_{j+1}\pi_{j+1} = \pi_j(\lambda_j + \mu_j)$ for j = 1, 2, ...

$\mu_1\pi_1 = \lambda_0\pi_0$ for j = 0

These comes about are known as the stream adjust conditions. You may take note that they propose that the rate at which moves happen into a specific state break even with the rate at which moves happen out of the same state. At this point, each steady-state likelihood can be decided by substituting in probabilities from lower states, appeared in more noteworthy detail on pages 1058-9 of the Winston content. Beginning with $\pi_1 = \pi_0\lambda_0\ \mu_1$, we can get the common equation

$\pi_j = \pi_0 c_j$

where

$c = \lambda_0\lambda_1...\lambda_{j-1}$

$\mu_1\mu_2...\mu_j$

2.6. Queueing Disciplines. It is simple for one to think of all lines working like a basic supply checkout line. That is to say, when an entry happens, it is included to the conclusion of the line and benefit is not performed on it until all of the entries that came some time recently it is served in the arrange, they arrived. In spite of the fact that this an exceptionally common strategy for lines to be dealt with, it is distant from the as it were way. The strategy in which entries in a line get prepared is known as the queueing teach. This specific illustration diagrams a first-come-first serve teach, or an FCFS teach. Other conceivable disciplines incorporate last-come-first-served or LCFS, and benefit in irregular arrange, or SIRO. Whereas the specific teach chosen will likely enormously influence holding up times for specific clients (no one needs to arrive early at an LCFS teach), the teach for the most part doesn't influence imperative results of the line itself, since entries are always accepting benefit regardless.

2.7. Kendall-Lee Documentation. Since depicting all of the characteristics of a line unavoidably gets to be exceptionally tedious, a much less complex documentation (known as Kendall-Lee documentation) can be utilized to depict a framework. Kendall-Lee documentation gives us six truncations for characteristics recorded in arrange isolated by slices. The to begin with and moment characteristics depict the entry and benefit forms based on their particular likelihood disseminations. For the to begin with and moment characteristics, M speaks to an exponential dispersion, E speaks to an Erlang dissemination, and G speaks to a common conveyance. The third characteristic gives the number of servers working together at the same time, moreover, known as the number of parallel servers. The fourth depicts the line teach by its given acronym. The fifth gives the most extreme number of number of clients permitted in the framework. The 6th gives the estimate of the pool of clients that the framework can draw from. For case, M/M/5/FCFS/20/ inf might speak to a bank with 5 tellers, exponential entry times, exponential benefit times, an FCFS line teach, a add up to capacity of 20 clients, and a boundless populace pool to draw from.

2.8. Little's Queueing Equation. In numerous lines, it is valuable to decide different holding up times and line sizes for Specific components of the framework in arrange to make judgments around how the framework ought to be run. Let us characterize L to be the normal number of clients in the line at any given minute of time accepting that the steady state has been come to. We can break that down into Lq, the

average number of clients holding up in the line, and Ls, the normal number of clients in benefit. Since clients in the framework can as it were either be in the line or in benefit, it goes to appear that L = Lq + Ls.

Likewise, we can characterize Was the normal time a client spends in the queueing framework. Wq is the normal sum of time went through in the line itself and Ws is the normal sum of time went through in benefit. As was the comparative case some time recently, W = Wq + Ws. It ought to be famous that all of the midpoints in the over definitions are the steady-state averages.

Defining λ as the entry rate into the framework, that is, the number of clients arriving the framework per unit of time, it can be appeared that

L = λW

L q = λWq

Ls = λWs This is known as Little's queueing equation [W 1062].

3. Queueing Models

With our establishment laid for the think about of vital characteristics of queueing frameworks, we can start to examine specific frameworks themselves. We will start by looking at one of the least difficult frameworks, the M/M/1/GD/∞/∞ system.

3.1. The M/M/1/GD/∞/∞ Queueing Framework. An M/M/1/GD/∞/∞ framework has exponential interarrival times, exponential benefit times, and one server. This framework can be modeled as a birth-death prepare where

λj = λ for (j = 0, 1, 2...)

μ0 = 0

μj = μ for (j = 1, 2, 3...)

Substituting this in to the condition for the steady-state likelihood, we get

πj = λjπ0

μj

We will characterize p = λ/μ as the activity escalated of the framework, which is a proportion of the entry and benefit rates. Knowing that the whole of all of the relentless state probabilities is break even with to one, we get

π0(1 + p + p2 + ... + pj) = 1

If we expect 0 ≤ p ≤ 1 and let the whole S = (1 + p + p2 + ... + pj), at that point S = 1 1-p and π0 = 1 - p. This yields

πj = pj(1 - p)

as the steady-state likelihood of state j [W 1058]. Note that if p ≥ 1, S approaches limitlessness, and in this way no unfaltering state can exist. Instinctively, if p ≥ 1, at that point it must be that λ ≥ μ, and if the entry rate is more noteworthy than the benefit rate, at that point the state of the framework will develop without end.

With the steady-state likelihood for this framework calculated, we can presently fathom for L. If L is the normal number of clients show in this framework, we can speak to it by the formula.

L =

∞ Xj =0

jπj = (1 - p)

∞ Xj =0

jpj

Let S = P∞ j=0 = p + 2p2 + 3p3 + ... Then pS = p2 + 2p3 + 3p4 + ... If we subtract,

we get

S - pS = p + p2 + p3... = p 1 - p

And S = p (1-p)2 . Replacing this into the equation for L will get us

L = (1 - p)   p (1 - p)2

= p 1 - p

=   λ μ - λ

To solve for Ls, we have to determine how many customers are in service at any given moment. In this particular system, there will always be one customer in service except for when there are no customers in the system. Thus, this can be calculated as

L q = 0π0 + 1(π1 + π2 + π3 + ...) = 1 - π0 = 1 - (1 - p) = p from here, Lq is an easy calculation.

L q = L - Ls = p

1 - p

\- p = p2

1 - p

Using Little's queueing formula, we can also solve for W, Ws, and Wq by dividing each of the corresponding L values by λ.

3.2. The M/M/1/GD/c/∞ Queueing Framework. An M/M/1/GD/c/∞ queueing framework has exponential interarrival and benefit times, with rates λ and μ separately. This framework is exceptionally comparative to the past framework, but that at whatever point c clients are display in the framework, all extra entries are avoided from entering, and are from there on no longer considered. For case, if a client were to walk up to a quick nourishment eatery and see that the lines were as well long for him to need to hold up there, he would go to another eatery instead.

A framework like this can be displayed as a birth-death prepare with these parameters:

λj = λ for j = 0, 1, 2, 3,…, c - 1

λc = 0 μ0 = 0

μj = μ for j = 1, 2, ..., c

The limitation λc = 0 is what sets this separated from the past framework. It makes it so that no state more prominent than c can ever be come to. Since of this limitation, a consistent state will continuously exist. This is since indeed if λ ≥ μ, there will never be more than c clients in the system.

Looking at equations determined from the think about of birth-death forms and once once more letting p = λ μ , we can determine the taking after steady-state probabilities [W 1068]:

π0 =

1 - p

1 - pc+1

πj = pjπ0 for j = 1, 2, ..., c πj = 0 for j = c + 1, c + 2, ..., ∞

A equation for L can be found in a comparable design, but is overlooked since of the chaotic calculations. The method is comparable to the one utilized in the past area. Calculating W is another issue. This is since in Little's queueing equation, λ speaks to the entry rate, but in this framework, not all of the clients who arrive will connect the line. In truth, λπc entries will arrive, but take off the framework. In this way, as it were λ - λπc = λ(1 - πc) entries will ever enter the framework. Substituting this into Little's queueing equation gives us

W = L λ(1 - πc)

3.3. The M/M/s/GD/∞/∞ Queueing Framework. An M/M/s/GD/∞/∞ queueing framework, like the prior framework we looked at, has exponential interarrival and profit times, with rates λ and μ. What sets this framework separated is that there are s servers willing to serve from a single line of clients, like maybe one would discover in a bank. If j ≤ s clients are display in the framework, at that point each client is being served. If j > s clients are in the framework, at that point s clients are being served and the remaining j - s clients are holding up in the line.

To demonstrate this as a birth-death framework, we have to watch that the passing rate is subordinate on how numerous servers are really being utilized. If each server completes benefit with a rate of μ, at that point the real passing rate is μ times the number of clients really being served. Parameters for this framework are as follows:

λj = λ for j = 0, 1, 2, .., ∞

μj = jμ for j = 0, 1, ..., s

μj = sμ for j = s + 1, s + 2, ..., ∞

In tackling the steady-state probabilities, we will characterize p = λ sμ . Take note that this definition too applies to the other frameworks we looked at, since in the other two frameworks, s = 1. The steady-state probabilities can be found in this framework in the same way as for other frameworks by utilizing the stream adjust conditions [W 1071]. I will to overlook these specific steady-state conditions since they are or maybe cumbersome.

3.4. The M/G/∞/GD/∞/∞ and GI/G/∞/GD/∞/∞ Queueing Frameworks. These frameworks are set separated in that they have an boundless number of servers, and in this way, a client never has to hold up in a line for their benefit to start. One way to think of this is as a self-service, like shopping on the web, for illustration. In this framework, it can be appeared that W = 1 μ

and L = λ μ

[W 1076]. It can too be appeared that the consistent state likelihood at state j is

πj =

(λ μ)je-(μλ)

j!

3.5. The Machine Repair demonstrate. The machine repair demonstrate is a M/M/R/GD/K/K line framework, where R is the number of servers, and K is both the measure of the client populace and the most extreme number of clients permitted in the framework. This show can clarify a circumstance where there are K machines that each break down at rate λ and R repair specialists who can each settle a machine at rate μ. This implies that both λ and μ are subordinate on either how numerous machines are remaining in the populace or how numerous repair laborers are in service.

Let's show this as a birth-death handle. Since λj depends on the number of machines cleared out in the populace that are not in benefit, we can say that-

$\lambda j = (K - j)\lambda$

We can calculate μj by looking at the number of repair specialists as of now in benefit. If a machine breaks down when all servers are active, it holds up in a line to be served. We can calculate μj as follows:

$\mu j = j\mu$ for j = 0, 1, 2, 3, .., R

$\mu j = R\mu$ for j = R + 1, R + 2, ..., K

The steady-state likelihood for a machine repair framework, inferred from page 1081 of the Winston content is

πj = Kj pjπ0 for J = 0, 1, ..., R

πj = Kj j!pjπ0

R!Rj-R for J = R + 1, R + 2, ..., K

3.6. The M/G/s/GD/s/∞ Queueing Framework. Another sensible demonstrate of a line is one where if a client arrives and sees all of the servers active, at that point the client exits the framework totally without getting benefit. In this case, no real line is ever shaped, and we say that the blocked clients have been cleared. Since no line is ever shaped, Lq = Wq = 0. If λ is the entry rate and 1/μ is the cruel benefit time, at that point W = Ws = 1/μ. In this framework, entries are turned absent at whatever point s clients are display, so πs is break even with to the division of all entries who are turned absent by the framework. This implies that an normal of λπs entries per unit of time will never enter the framework, and in this way, λ(1 - πs) entries per unit of time will really enter the framework. This leads us to the conclusion based on Little's queueing equation that L = Ls = λ(1-πs) μ .

4. Field Research

Trying to put this information of queueing hypothesis to a few utilize, I took a trip to a neighbourhood Bank of America to ponder their lines and see if their framework seem be fitted to a specific demonstrate. The Bank of America framework demonstrated to be a basic one to keep track of especially since all of the clients held up in a single line instep of a few partitioned ones. That made it conceivable to precisely keep track of how long each client had been holding up in line. Be that as it may, the number of servers on obligation amid the period of think about kept changing, and to keep the demonstrate from getting to be overcomplicated, a sensible esteem for s had to be chosen. There were 77 clients kept track of with a chosen esteem of 5 servers over the course of one hour of observation.

On normal, a client arrived each 46 seconds, or each 0.77 minutes. Hence, the test interarrival rate was 1.30 clients per diminutive. Too, benefit completions took a normal of 146 seconds, or 2.44 minutes. This implies that the test benefit rate was 0.41 benefit completions per diminutive. The normal holding up time in the line was 43.6 seconds, or 0.73 minutes. The normal add up to time a client went through in the framework was 3.17 minutes. In a perfect world, this framework may be demonstrated as a M/M/5/FCFS/∞/∞ system.

It is vital to keep in mind that a framework such as this one is accepted to take after exponential dispersions for interarrival times and benefit times. Setting λa = 1.30 and λs = 0.41, the conceivable exponential disseminations for the interarrival and benefit times individually were

f(t) = 1.3e-1.3t and f(t) = .41e-.41t

In arrange to see if these conveyances fit suitably with the information, they were subjected to a chi-square goodness of fit test, the strategy of which can be found on page 1091 of the Winston content. For the interarrival times, the test come about with χ2(obs) = 2.026, which was too sufficient to conclude that the exponential dispersion can appropriately demonstrate the interarrival times. In any case, for the benefit times, the test brought about in χ2(obs) = 36.57, which was well out of the run for the benefit times to be legitimately displayed by the exponential conveyance. It appears that this might be a result of utilizing a generally little test size.

Chi-square tests were performed with other conveyances as well, but nothing appeared to fit sensibly. In this way, we were incapable to completely interface this consider with one of the already examined models. If information had been taken from the bank for a longer period of time, it appears likely that exceptions would have had lesser impact on the chi-square tests. Given that the entries fit the exponential conveyance, benefit times likely seem have been exponential as well. To affirm that would require advance observation.

5. Conclusion

With the information of likelihood hypothesis, input and yield models, and birth passing forms, it is conceivable to infer numerous diverse queueing models, counting but not constrained to the ones we watched in this paper. Queueing hypothesis can be pertinent in numerous genuine world circumstances. For illustration, understanding how to show a multiple-server line seem make it conceivable to decide how numerous servers are really required and at what wage in arrange to maximize budgetary effectiveness. Or maybe a queueing show seem be utilized to think about the life expectancy of the bulbs in road lights in arrange to way better get it how regularly they require to be replaced.

The applications of queueing hypothesis expand well past holding up in line at a bank. It may take a few inventive considering, but if there is any sort of situation where time passes some time recently a specific occasion happens, there is likely a few ways to create it into a queueing demonstrate. Lines are so commonplace in society that it is exceedingly beneficial to consider them, indeed if as it were to shave a few seconds off one's hold up in the checkout line.

6. References

1. [W] Wayne L Winston, Operations Inquire about: Applications and Calculations, 2nd version, PWS-Kent Distributing, Boston, 1991.

2. Agrawal, Subhash Chandra (1985). Metamodeling: A Study of Approximation in Queueing Models. MIT Press, Cambridge, MA. (262pp.)

3. Ash, Robert B. (1975). Topics in Stochastic Processes. Academic Press, New York.

4. Asmussen, Soren (1987). Applied Probability and Queues. John Wiley and Sons, Chichester (318pp. ISBN 0-471-91173-9)