IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal
International Research Journal IJNRD Research Through Innovation

Wormholes in spacetime and there use in interstellar travel

Jatin Tomar

Intro and summary

Black holes are not usable for interstellar travel

- Horizon of black hole is surface separating interior trapped regions of hole
- If person of height L goes there, it experiences a relative acceleration of magnitude nearly $L(\frac{2GM}{c^3})^{-2}$~(10 Earth gravities)$(\frac{L}{1\,m})(\frac{M}{10^4}\ solar\ masses)^{-2}$ this acceleration is between head and feet of person.
- Black hole horizon is one way membrane, things may fill up but can't escape, hence the other side might be something weird to throw matter example a hypothetical white hole.
- White holes may possess an anti horizon , these are highly unstable against small perturbation. If an anti horizon is somewhere to form, a stray wave packet of light with arbitrarily small energy falling toward it would become more and more blueshifted and more and more energetic as it falls. By this exponentiating energy the wave packet would turn it into a normal horizon by sealing of the white hole.
- According to Kerr metric, rotating black holes possess in it's interior, pathways to another asymptotically flat regions of spacetime.
- Once a newborn rotating hole settles down to time independent state, it must have Kerr form, which is only applicable to outside of horizon. There's no reason expect stellar collapse to form a Kerr hole to also form a Kerr interior.
- When a wave packet approaches Cauchy horizon(anti horizon), it's energy will create exponentially growing tidal gravitational fields that may seal off the tunnel and convert it into a physical singularity. Thus interior of black hole possess not tunnels but rather singularities of near infinitely strong tidal gravitational fields.
- If Kerr tunnels somehow stabilizes, it would process ring shaped singularities. If physics were totally classical and if hole were sufficiently massive and raid rotating, a traveller can be through. But due to predictions of QFT, singularities by breaking down the vacuum should spew an intense flux of high energy particles into tunnel which may kill the traveller and may seal off the passage.

Schwarzschild wormholes are not traversable

- Worm holes are solutions of Einstein's field equations ,there are some objections though to travel through wormholes.
    - Tidal gravitational forces at throat has same magnitude as of black hole. They're so large that unless wormhole's mass exceeds 10^4 solar masses so its throat circumference exceeds 10^5 km, traveller certainly dies.
    - Schwarzschild wormholes are dynamic in nature, as time passes it expands from 0 to max to 0 throat circumference which is so rapid that nothing can escape.

Desired properties of a traversable wormhole

- Metric should be both symmetric and static, wormhole might be unstable for spherical and non spherical perturbation.

- Solution must obey Einstein's field equations everywhere.
- To be a wormhole, solution must have a throat that connects two asymptotically flat regions of spacetime.
- There should be no horizon.
- Tidal gravitational forces must be bearable.
- Traveller must able to cross through in finite and small proper time as measured by both him and observers on other side.
- Solutions must be perturbatively stable.
- Matter and fields which generates wormhole spacetime curvature must have physically reasonable stress energy tensor.

Mathematical details of traversable wormholes

Form of metric

$$ds^2 = -e^{-2\psi}c^2dt^2 + \frac{dr^2}{(1-\frac{b}{r})} + r^2(d\theta^2 + sin^2\theta d\phi^2) \quad (1)$$

$$\phi = \phi(r)\ is\ shape\ function$$

$$b = b(r)\ is\ redshift\ function$$

$$2\pi r\ is\ throat\ circumference$$

r is a non monotonic (r decrease from ∞ *to* b_0 *while getting closer to throat then increase while going away from throat from* b_0 *to* ∞)

Equations of structure for wormhole

1. Riemann, Ricci and Einstein tensors

In order to study tidal forces felt by traveller crossing the hole, Riemann and Einstein tensors are needed

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta \qquad (2)$$

$$x_0 = ct$$

$$x^1 = r$$

$$x^2 = \theta$$

$$x^3 = \phi$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}g^{\alpha\gamma}\left(g_{\lambda\beta,\gamma} + g_{\lambda\gamma,\beta} - g_{\beta\gamma,\lambda}\right)\ christoffel\ symboles \qquad (3)$$

$$R^\alpha_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\lambda\gamma}\Gamma^\lambda_{\beta\delta} - \Gamma^\alpha_{\lambda\delta}\Gamma^\lambda_{\beta\gamma},\qquad Riemann\ curveture\ tensor \qquad (4)$$

Here comma denotes a partial derivative($g_{\alpha\beta,\gamma} = \frac{\partial g_{\alpha\beta}}{\partial x^\gamma}$)

Applying these equations to metric (1),

$$R^t_{rtr} = -R^t_{rrt} = \left(1-\frac{b}{r}\right)^{-1} e^{-2\Phi}R^r_{ttr}$$

$$= -\left(1-\frac{b}{r}\right)^{-1} e^{-2\Phi}R^r_{trt}$$

$$= -\Phi'' + (b'r-b)[2r(r-b)]^{-1}\Phi' - (\Phi')^2$$

$$R^t_{\theta t\theta} = -R^t_{\theta\theta t} = r^2e^{-2\Phi}R^\theta_{tt\theta} = -r^2e^{-2\Phi}R^\theta_{t\theta t}$$

$$= -r\Phi'\left(1-\frac{b}{r}\right)$$

$$R^t_{\phi t\phi} = -R^t_{\phi\phi t} = r^2e^{-2\Phi}sin^2\theta R^\phi_{tt\phi}$$

$$= -r^2 e^{-2\Phi} sin^2\theta R^{\phi}_{t\phi t}$$

$$= -r\Phi'(1-\frac{b}{r})sin^2\theta$$

$$R^{r}_{\theta r\theta} = -R^{r}_{\theta\theta r} = -r^2\left(1-\frac{b}{r}\right)R^{\theta}_{rr\theta}$$

$$= r^2\left(1-\frac{b}{r}\right)R^{\theta}_{r\theta r}$$

$$= \frac{(b'r-b)}{2r}$$

$$R^{r}_{\phi r\phi} = -R^{r}_{\phi\phi r} = -r^2\left(1-\frac{b}{r}\right)sin^2\theta R^{\phi}_{rr\phi}$$

$$= r^2\left(1-\frac{b}{r}\right)sin^2\theta R^{\phi}_{r\phi r}$$

$$= \frac{(b'r-b)sin^2\theta}{2r}$$

$$R^{\theta}_{\phi\theta\phi} = -R^{\theta}_{\phi\phi\theta} = sin^2\theta R^{\phi}_{\theta\phi\theta} = -sin^2\theta R^{\phi}_{\theta\theta\phi}$$

$$= \frac{b}{r}sin^2\theta$$

Above equations are (5)

Here prime denotes derivative with respect to radial coordinate r and basis vectors ($e_t, e_r, e_\theta, e_\phi$) associated with coordinate system (ct,r,θ, ϕ) vector separation between two events with coordinate separation ($\Delta t, \Delta r, \Delta\theta, \Delta\phi$) is

$$\Delta s = c\Delta t e_t + \Delta r e_e + \Delta\theta e_\theta + \Delta\phi e_\phi$$

[In notations preferred by differential geometers, $e_t = c^{-1}\frac{\partial}{\partial t}$ $e_r = \frac{\partial}{\partial r}$ $e_\theta = \frac{\partial}{\partial \theta}$ $e_\phi = \frac{\partial}{\partial \phi}$]

In coordinate system (r, θ, ϕ $constant$) (6)

$$e_{t`} = e^{-\Phi} e_t$$

$$e_{r`} = (1-\frac{b}{r})^{\frac{1}{2}} e_r$$

$$e_{\theta`} = r^{-1} e_\theta$$

$$e_{\phi`} = (rsin\theta)^{-1} e_\phi$$

In special relativity forms, (7)

$$g_{\alpha`\beta`} = e_{\alpha`} e_{\beta`} = \eta_{\alpha`\beta`} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore simplified Riemann tensor, (8)

$$R^{t`}_{r`t`r`} = -R^{t`}_{r`r`t`} = R^{r`}_{t`t`r`} = -R^{r`}_{t`r`t`}$$

$$= (1-\frac{b}{r})\{-\Phi" + (b'r-b)[2r(r-b)]^{-1}\Phi' - (\Phi')^2\}$$

$$R^{t`}_{\theta`t`\theta`} = -R^{t`}_{\theta`\theta`t`} = R^{\theta`}_{t`t`\theta`} = -R^{\theta`}_{t`\theta`t`} = \frac{-\left(1-\frac{b}{r}\right)\Phi'}{r}$$

$$R^{t`}_{\phi`t`\phi`} = -R^{t`}_{\phi`\phi`t`} = R^{\phi`}_{t`t`\phi`} = -R^{\phi`}_{t`\phi`t`} = \frac{-(1-\frac{b}{r})\Phi'}{r}$$

$$R^{r`}_{\theta`r`\theta`} = -R^{r`}_{\theta`\theta`r`} = R^{\theta`}_{r`\theta`r`} = -R^{\theta`}_{r`r`\theta`} = \frac{(b'r-b)}{2r^3}$$

$$R^{r`}_{\phi`r`\phi`} = -R^{r`}_{\phi`\phi`r`} = R^{\phi`}_{r`\phi`r`} = -R^{\phi`}_{r`r`\phi`} = \frac{(b'r-b)}{2r^3}$$

$$R^{\theta`}_{\phi`\theta`\phi`} = -R^{\theta`}_{\phi`\phi`\theta`} = R^{\phi`}_{\theta`\phi`\theta`} = -R^{\phi`}_{\theta`\theta`\phi`} = \frac{b}{r^3}$$

From here we can contract Riemann tensor and calculate scalar curvature R and Ricci tensor $R_{\mu`v`}$

$$R_{\mu`v`} = R^{\alpha`}_{\mu`\alpha`v`} \qquad (9)$$

$$\mathrm{R}{=}g^{\mu`v`}R_{\mu`v`} \qquad (10)$$

Therefore we can get Einstein tensor and hence we enter into Einstein filed equations

$$G_{\mu`v`} = R_{\mu`v`} - \tfrac{1}{2}Rg_{\mu`v`} \qquad (11)$$

Non zero components of Einstein tensors are (12)

$$G_{t`t`} = \frac{b'}{r^2}$$

$$G_{r`r`} = \frac{-b}{r^3} + \frac{2\left(1-{}^{b}/_{r}\right)\Phi'}{r}$$

$$G_{\theta`\theta`} = G_{\phi`\phi`} = (1-\frac{b}{r})[\Phi" - \frac{b'r-b}{2r(r-b)}\Phi' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r-b}{2r^2(r-b)}]$$

2. Stress energy tensor

As per Birkhoff, only one kind of vacuum, spherical wormhole is allowed by Einstein field equations which are non traversable . thus traversable wormholes threaded by matter or fields with non zero stress energy tensor.

Einstein's field equations requires stress energy tensor to be proportional to Einstein tensor, hence $T_{\mu`v`}$ has same algebraic structure as $G_{\mu`v`}$ of eq(12), hence non zero components must be $T_{t`t`}$,$T_{r`r`}$,$T_{\theta`\theta`} = T_{\phi`\phi`}$

$$T_{t`t`} = \rho(r)c^2 \qquad (set\ 13)$$

$$T_{r`r`} = -\tau(r)$$

$$T_{\theta`\theta`} = T_{\phi`\phi`} = p(r)$$

$\rho(r)$ is total density of mass energy measured $({}^{g}/_{cm^3})$

τ(r) is tension per unit area measure in radial direction [it is negative of radial pressure with units ${}^{dyn}/_{cm^2}$]

P(r) is pressure in $({}^{dyn}/_{cm^2})$ in lateral directions.

The stress energy tensor of an ordinary "perfect fluid" is special case of (13) ;it has $-\tau = p$.

Another special case is radially pointing electric field of strength E(r) for which $\tau = p = \rho c^2 = E^2/8\pi$.

3. Einstein field equations

The Einstein's field equations ,

$$G_{\alpha`\beta`} = 8\pi G c^{-4} T_{\alpha`\beta`}$$

By manipulating equations (12) and (13)

$$b' = 8\pi G c^{-2} r^2 \rho \qquad (14)$$

$$\Phi' = \frac{(-8\pi G c^{-4} \tau r^3 + b)}{[2r(r-b)]} \qquad (15)$$

$$\tau' = (\rho c^2 - \tau)\Phi' - \frac{2(p+\tau)}{r} (16)$$

(14) and (15) are temporal and radial parts of field equations and (16) lateral (θ, ϕ) parts of field equations with Φ" eliminated using radial derivative of (15).

Physical interpretation of (16) is hydrostatic equilibrium for material threading wormhole, forces due to radial tension gradient , the lateral "Roman arch" pressure and gravitational pull.

The redundancy between Einstein field equations and law of force balance is deep aspect of general relativity.

Equations 14 to 16 are 3 differential equations relating 5 unknown functions of r: b,Φ, ρ, τ, p.

To solve these equations we need to need to assume specific field or matter as stress energy tensor source to derive "equations of state", for radial tension as function of mass energy density $\tau(\rho)$ and lateral pressure as $p(\rho)$, these equations of state and 3 differential equations form 5 equations for 5 unknowns.

From equations (14) to (16)

$$\rho = \frac{b'}{8\pi G c^{-2} r^2} \quad (17)$$

$$\tau = \frac{[\frac{b}{r} - 2(r-b)\Phi']}{[8\pi G c^{-4} r^2]} \qquad (18)$$

$$p = \frac{r}{2}[(\rho c^2 - \tau)\Phi' - \tau'] - \tau \quad (19)$$

equation (17) and our choice of b(r) gives us $\rho(r)$ equation (18) and our choices for both b(r) and $\Phi(r)$ will yield us $\tau(r)$ and finally equation (19) together with above all will give us p(r).

4. Boundary conditions

We wish to let stress energy generates curvature extent out to arbitrarily large radii, or we may confine it to interior of sphere of surface radius r=R_s, we may require ρ, τ *and* p all vanishes at r>R_s

In this case we need $\tau \longrightarrow 0$ but ρ and p remain finite in $\lim_{r \longrightarrow R_s} from\ below$ (20)

Equation (14) to (16) evaluated outside r=R_s constraint external spacetime geometry to have general Schwarzschild form

$b(r) = b(R_s) = constant \equiv B$ at $r > R_s$ (21)

$\Phi(r) = \frac{1}{2}\ln(1 - \frac{B}{r})$ $at\ r > R_s$ (22)

If there's no cutoff in stress energy, we shall still require that field die out fast enough radially that spacetime is asymptotically flat

$$\frac{b}{r} \longrightarrow 0\ and\ \Phi \longrightarrow 0\ as\ r \longrightarrow \infty \quad (23)$$

Spatial geometry of wormhole

1. Mathematics of embedding

Putting $\theta = \frac{\pi}{2}$ and t⟶ 0 in equation (1)

$$ds^2 = \left(1 - \frac{b}{r}\right)^{-1} dr^2 + r^2 d\phi^2 \quad (24)$$

We wish to construct, in 3D Euclidian space a 2D surface with same geometry as this slice.

For this we need to introduce cylindrical system

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2 \quad (25)$$

Embedded surface will be axially symmetric and can be described as z=z(r)

$$ds^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\phi^2 \quad (26)$$

Equation (24) and (26) are similar if we identify both equations, and if we require z=(r) which describes embedded surface in (24), to satisfy

$$\frac{dz}{dr} = \pm\left(\frac{r}{b(r)} - 1\right)^{\frac{-1}{2}} \quad (27)$$

Equation (27) displays manner in which b=b(r) shapes wormhole's geometry

2. Schwarzschild Wormhole

Consider a Schwarzschild wormhole hole of radius r=B =constant, embedded surface (solution of (27)) becomes

$$z(r) = \pm 2B(\frac{r}{B} - 1)^{\frac{1}{2}} \quad (28)$$

Here throat radius is at r=B hence $\frac{dz}{dr}$ becomes infinite corresponding to a vertical slope of embedded surface although this is true for every wormhole not exclusively for Schwarzschild wormholes.

Because of divergence of $\frac{dz}{dr}$, r is not a good coordinate to use throat vicinity. Much better is proper radial distance measured by static observers

$$dl = \pm\left(1 - \frac{B}{r}\right)^{\frac{-1}{2}} dr \quad (29a)$$

That is,

$$l = \pm\left[\sqrt{r(r-B)} + B\ln\left(\sqrt{\frac{r}{B}}\right) + \sqrt{\frac{r}{B} - 1}\right] \quad (29b)$$

Here l is radial distance, when it is +ve sign in eq(s) 29, it shows upper universe and when -ve sign shows, lower universe. Very far from Schwarzschild throat embedding surface become flat.

$$\frac{dz}{dr}(l \rightarrow \pm\infty) = 0 \quad (30)$$

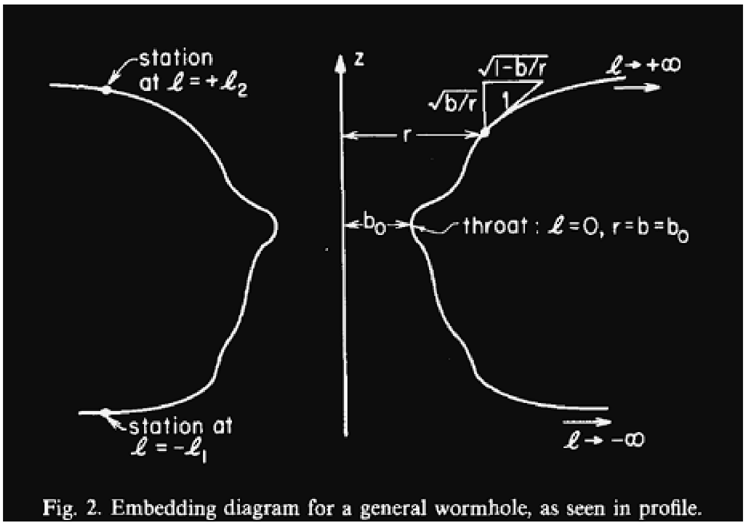

Fig. 2. Embedding diagram for a general wormhole, as seen in profile.

3. General Wormhole

Every wormhole should have minimum radius r=b_0 at which embedded surface is vertical or at which (27) diverges or at which b(r)=r, there exist a minimum value o r=b_0 in wormhole and at r=b_0, b=b_0 (31). Generally radial coordinate r ill behaved near throat but proper radial distance must well behave everywhere or we must require l(r) finite throughout spacetime (33).

$$l = \pm \int_{b_0}^{\tau} \frac{dr}{\left[1 - \frac{b(r)}{r}\right]^{\frac{1}{2}}} \quad (32)$$

Which also implies $1 - \frac{b(r)}{r} > 0 \quad (34)$ throughout spacetime

Far from throat, in both directions ,space must be asymptotically flat that is,

$\frac{dz}{dr} = \pm \left(\frac{r}{b} - 1\right)^{\frac{-1}{2}} \to 0 \ \ as\ l \to \pm\infty$ that is, $\frac{b}{r} \to 0asl \to \pm\infty \quad (35)$

Since (27) and (32) imply that for embedded wormhole

$$\frac{dz}{dl} = \pm\sqrt{\frac{b}{r}} and \frac{dr}{dl} = \pm\sqrt{1 - \frac{b}{r}} \quad (36)$$

Above figure depicts somewhat general wormhole shape and geometrical meanings of (36)

4. Absence of horizon

- Horizons are physically non singular surfaces at which $g_{00} \equiv -e^{2\Phi} \to 0$ (vanishing proper time lapse during any finite coordinate time)
- Demand that our traversable wormhole not possess any horizons corresponds to
  $\Phi(r)\ is\ finite\ every\ where \quad (37)$

5. Tidal gravitational forces and time to traverse the wormhole

Let a traveller journeys radially throughout wormhole starting at station in lower universe at l=-l_1 ending at station In upper universe at l=+l_2 , let v(r) be radial velocity through radius r measured by static observer, proper time elapsed as by traveller be $d\tau_T$,

$$v = \frac{dl}{e^{\Phi}dt} = \mp \frac{dr}{\left(1 - \frac{b}{r}\right)^{\frac{1}{2}} e^{\Phi}dt} \quad (38a)$$

$$v\gamma \equiv \frac{v}{\left[1 - \left(\frac{v}{v}\right)^2\right]^{\frac{1}{2}}} = \frac{dl}{d\tau_T} = \mp \frac{dr}{\left(1 - \frac{b}{r}\right)^{\frac{1}{2}} d\tau_T} \quad (38b)$$

Here (-) for first half or lower universe and (+) for second half or upper universe, note that trip start and end at rest.

$$v = 0 \; at \; l = \mp l$$

$$v > 0 \; at - l < l < +l \quad (set39)$$

Stations at l=-l and l=+l must be far enough from throat so that gravitational effects experience by observers must me least.

$$\frac{b}{r} \ll 1, \qquad |\Phi| \ll 1$$

$$\left|\Phi'^{c^2}\right| < \sim g \; at \; l = -l_1 \; and \; l = +l_2 (40)$$

Because |Φ| << 1 at stations, proper time equals to spacetime metric in (1)

For human convenience in a wormhole, 3 conditions must be satisfied,

- Entire trip should require less than 1 year (observed by traveller and static observers)

$$\Delta\tau_1 = \int_{-l_1}^{l_2} \frac{dl}{v\gamma} < \sim 1 \; year \quad (41a)$$

$$\Delta t = \int_{-l_1}^{l_2} \frac{dl}{ve^{\Phi}} < \sim 1 \; year \quad (41b)$$

- Acceleration felt by traveller must not exceed by earth's gravity.
- Tidal acceleration between body parts must not exceed by earth's gravity.

Let us define orthonormal basis for frame of traveller be $(e_{0`}, e_{1`}, e_{2`}, e_{3`})$ and from frame of static observers be $(e_{t`}, e_{r`}, e_{\theta`}, e_{\phi`})$ therefore by Lorentz transformation,

(set 42)

$$e_{0`} = u = \gamma e_{t`} \mp \gamma \left(\frac{v}{c}\right) e_{r`}$$

$$e_{1`} = \mp\gamma e_{r`} + \gamma \left(\frac{v}{c} e_{t`}\right)$$

$$e_{2`} = e_{\theta`}$$

$$e_{3`} = e_{\phi`}$$

Here u is traveller's 4 velocity, $e_{1`}$ is in increasing direction of l.

The four acceleration experienced by traveller's body is always orthogonal to four velocity since the traveller is moving radially, so acceleration must be radial so $a_{2`}=a_{3`}=0$ and $a_{1`}=ae_{1`}$.

Easy way to compute a to regard u_α as function of traveller's radial location r to evaluate,

$\frac{a_t}{c^2} = u_{t;\alpha}u^\alpha = u_{t,r}u^r - \Gamma_{\alpha t\beta}u^\alpha u^\beta$ in (ct, r,θ, ϕ) then from (6),(7),(42) we get

$$a = \mp\left(1 - \frac{b}{r}\right)^{1/2} e^{-\Phi}\left(\gamma e^{\Phi}\right)'^{c^2} = e^{-\Phi}\frac{d}{dl}\left(\gamma e^{\Phi}\right)c^2 \quad (43)$$

Since we want our traveller not to feel acceleration larger than earth's gravity then,

$$\left|e^{-\Phi}\frac{d\left(\gamma e^{\Phi}\right)}{dl}\right| \leq \frac{g}{c^2} \approx \frac{1}{0.97 \; 1 \; yr} \qquad (44)$$

Let ξ be tidal gravitational forces experienced by traveller between her head and feet which is purely spatial such that $\xi \cdot u = 0$ u is four velocity then tidal acceleration felt by her is

$$\Delta a^{\alpha`} = -c^2 R^{\alpha`}_{\beta`\gamma`\delta`} u^{\beta`} \xi^{\gamma`} u^{\delta`} \quad (45)$$

Here $R^{\grave{\alpha}}_{\grave{\beta}\grave{\gamma}\grave{\delta}}$ are components of Riemann curvature tensor, RHS is tidal acceleration of freely falling two test particles separated by distance ξ with four velocity u. since $u^{\grave{\alpha}} = \delta^{\grave{\alpha}}_{\grave{0}}$ and $\xi^{\grave{0}} = 0$ in traveller's frame and since Riemann curvature tensor is asymmetric in first two indices of, $\Delta a^{\grave{\alpha}}$ is purely special with components,

$$\Delta a^{\grave{j}} = c^2 R^{\grave{i}}_{\grave{0}\grave{k}\grave{0}} = -c^2 R_{\grave{j}\grave{0}\grave{k}\grave{0}} \xi^{k'} \quad (46)$$

By transforming components of (8) from observer 's to traveller's frame we get

$$R_{\grave{1}\grave{0}\grave{1}\grave{0}} = R_{\grave{r}\grave{t}\grave{r}\grave{t}} = -(1-\frac{b}{r}) \times (-\Phi" + \frac{b'r-b}{2r(r-b)}\Phi' - (\Phi')^2 \qquad (47a)$$

$$R_{\grave{2}\grave{0}\grave{2}\grave{0}} = R_{\grave{3}\grave{0}\grave{3}\grave{0}} = \gamma^2 R_{\grave{\theta}\grave{t}\grave{\theta}\grave{t}} + \gamma^2 \left(\frac{v}{c}\right)^2 R_{\grave{\theta}\grave{r}\grave{\theta}\grave{r}} = \frac{\gamma^2}{2r^2}\left[\left(\frac{v}{c}\right)^2 \left(b' - \frac{b}{r}\right) + 2(r-b)\Phi'\right] \quad (47b)$$

Here tidal acceleration (46) takes a simple form

$$\Delta a^{\grave{1}} = -c^2 R_{\grave{1}\grave{0}\grave{1}\grave{0}} \xi^{\grave{1}}, \Delta a^{\grave{2}} = -c^2 R_{\grave{2}\grave{0}\grave{2}\grave{0}} \xi^{\grave{2}}, \Delta a^3 = R_{\grave{3}\grave{0}\grave{3}\grave{0}} \xi^3 \quad (48)$$

Combining (47) and (48) we get

$$|R_{\grave{1}\grave{0}\grave{1}\grave{0}}| = |(1-\frac{b}{r})(-\Phi" + \frac{b'r-b}{2r(r-b)}\Phi' - (\Phi)^2)| \leq \frac{g}{c^2 \times 2m} \cong \frac{1}{(10^2\ cm)^2} \qquad (49)$$

$$|R_{\grave{2}\grave{0}\grave{2}\grave{0}}| = \left|\frac{\gamma^2}{2r^2}\left[\left(\frac{v}{c}\right)^2 \left(b' - \frac{b}{r}\right) + 2(r-b)\Phi'\right]\right| \leq \frac{g}{c^2 \times 2m} \cong \frac{1}{(10^2 cm)^2} \quad (50)$$

(49) is tidal radial constraint, can be satisfied by $\Phi' = 0$ and (50) is lateral tidal constraint, constraining speed v through which traveller crosses the worm hole.

Stability of Wormhole

Stability of wormhole depends upon followings

- Throat stability
- Exotic matter
- Quantum effects
- Hawking radiation

Assembly of Wormhole

This assembly might be daunting because it entails change in topology of space in General theory of relativity, such changes will entail singularities which can be understood only if gravity is quantized.

Conclusion

Wormhole solutions to Einstein's equations presented is not only tool for teaching GTR. Today they are also an intriguing possibility for actual construction by advanced civilisations. However, any hope that they might be constructable must rely on future discovery of an exotic field or quantum state of known fields with tension that exceeds energy density on macroscopic scales. We must keep in mind that such exotic field might eventually be ruled out on fundamental microphysical grounds, and that such an exclusion would prevent wormhole by flat. Moreover, even if an exotic field were available, several other difficulties might prevent construction of a real traversable wormhole: the topology change required for wormhole formation may not classically allowed, not even quantum mechanically understood and might be quantum mechanically forbidden. Existing exotic matter might interact only very strongly with ordinary matter preventing human travel because of overbearing field stresses. And backward time travel that appears to be permitted by such wormholes might, in some as yet unimaginable way prevent their construction.

Acknowledgements

"Wormholes in spacetime and their use for interstellar travel" by Micheal S Morris and Kip Throne,

Theoretical Astrophysics , California Institute of Technology, Pasadena, California 91125