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GEOMETRICALLY PROOF OF THE RULES OF DIFFERENTIATION

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INTRODUCTION

Prior to taking part in this Mathematics Research Project, I have seen and go through the details of proof of various rules of differentiation they include assumptions.

With greatly respect to Father **Gottfried Wilhelm von Leibniz (1646 -1716)** , I am working-hard to find out the differentiation rules from represented geometrically diagrams. Before I begin to talk about the project itself, I would to say that I really enjoyed working on my Project.

Abstract:-

In this paper $f(x)$ and $q(x)$ and $\frac{1}{q(x)}$ are considered as the breadth and length of the closed curves or Rectangle and h as the increment of each function .Each figure is shown for every consideration .

Contents:-

GEOMETRICALLY PROOF OF THE FOLLOWING RULES OF DIFFERENTIATION

1. **Quotient rule :**

For, $y = \frac{f(x)}{q(x)}$

Then , $\frac{dy}{dx} = \frac{q(x)\frac{d\,f(x)}{dx} - f(x)\frac{d\,q(x)}{dx}}{(q(x))^2}$

Or, $\frac{dy}{dx} = \frac{q(x)\times f'(x) - f(x)\times q'(x)}{(q(x))^2}$

Proof :-

Step :1: -

<u>Finding the area of a Rectangle ABCD from Fig:1.1</u>:-

Consider $f(x)$ as the breadth of a rectangle ABCD and $q(x)$ as the length of a rectangle ABCD as shown in fig:1.1.

{Where, $0 < q(x) \le 1$; meaning that $\boldsymbol{q(x) is\ greater\ than\ zero\ but\ less\ than\ or\ equal\ to\ 1}$}.

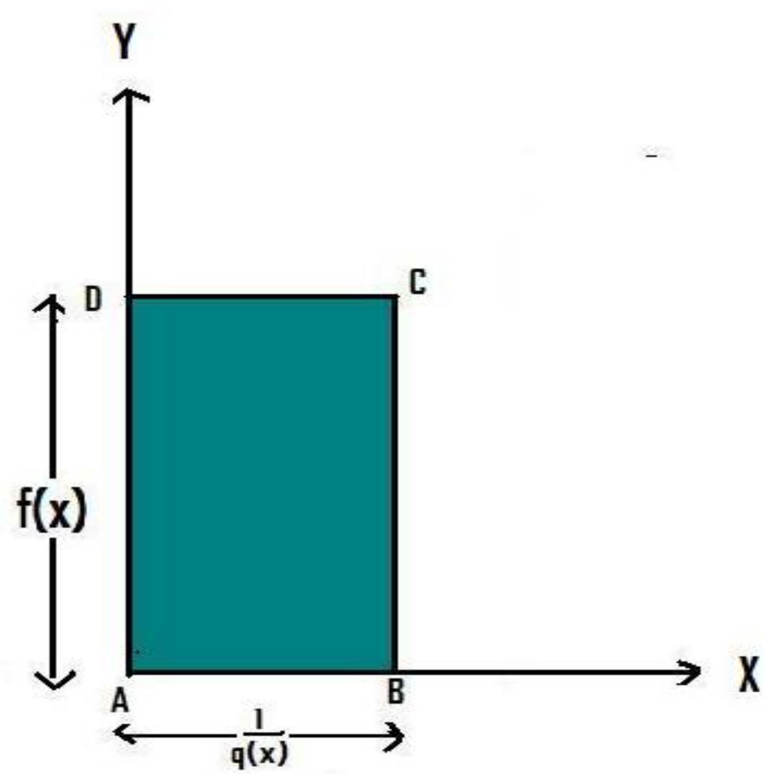

Fig:1.1

From a Rectangle ABCD (In Fig:1.1), we have ,

Length AB $= f(x)$ ------------------------------------ (1.0)

Breadth AD $= \frac{1}{q(x)}$ ----------------------------------- (1.1)

∴ Area of Rectangle ABCD = Length AB × Breadth AD

Area of Rectangle ABCD $= \frac{1}{q(x)} \times f(x)$

Area of Rectangle ABCD = $\frac{f(x)}{q(x)}$ ------------------------- (1.2)

Step:2 :-

Introducing an increment h to both functions $f(x)$ and (x) :

If the function $f(x)$ has the increment h , then the function $q(x)$ will also have an increment h , as shown below in the fig.1.2

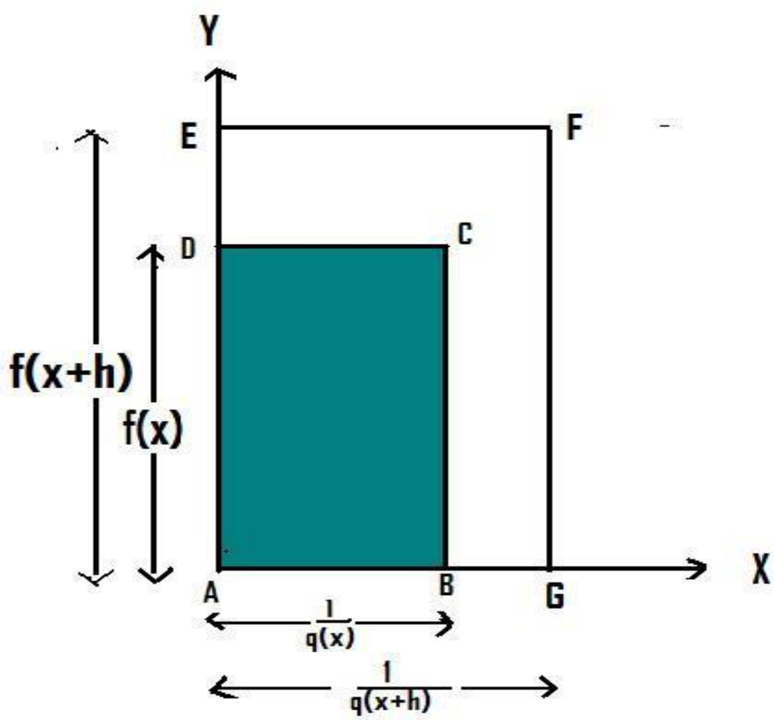

Fig:1.2

From a Rectangle AGFE (Fig:1.2), we have

Length AG $= \frac{1}{q(x+h)}$ ------------------------ (1.3)

Breadth AE = $f(x+h)$ ------------------------- (1.4)

∴ Area of Rectangle AGFE = Length AG × Breadth AE

Area of Rectangle AGFE = $\frac{1}{q(x+h)} \times f(x+h)$

Area of Rectangle AGFE= $\frac{f(x+h)}{q(x+h)}$------------------------- (1.5)

Step.3:

Finding the Area of the two new Rectangles (Rectangle BGOC and Rectangle DOFE) :-

After Producing a line OC as shown in Fig:1.3 , then the lengths , breadths and areas of the two new Rectangle DOFE and Rectangle BGCO can be find out .

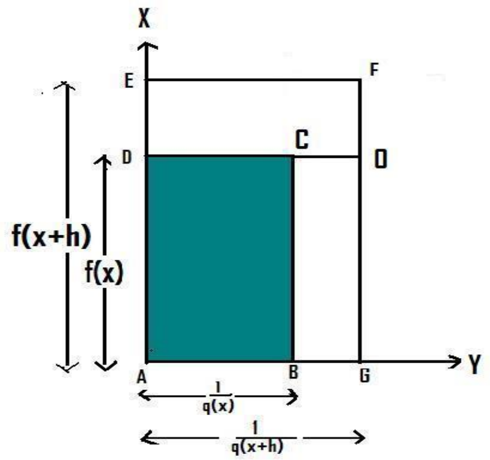

Fig:1.3

From the above fig:1.3 ,We have

Length of a Rectangle BGOC = Length of a Rectangle AGFE − Length of a Rectangle ABCD

Length of a Rectangle BGOC = AG − AB

Length of a Rectangle BGOC = $\frac{1}{q(x+h)} - \frac{1}{q(x)}$ -------------------------------- (1.6)

And

Breath of a Rectangle BGOC = Breadth of a Rectangle ABCD

Breath of a Rectangle BGOC =$f(x)$ -- (1.7)

∴ **Area of a Rectangle BGOC** = length of a Rectangle BGFO × Breadth of a Rectangle BGFO

Area of a Rectangle BGOC = $\left\{\frac{1}{q(x+h)} - \frac{1}{q(x)}\right\} \times f(x)$ {Using (1.6) and (1.7)

Area of a Rectangle BGOC = $f(x)\left(\frac{1}{q(x+h)} - \frac{1}{q(x)}\right)$ ---------------------------- (1.8)

Also,

From the above fig:1.3 , We have

Length of a Rectangle DOFE =Length of a Rectangle ABFE

Length of a Rectangle DOFE = AG

∴ Length of a Rectangle DOFE =$\frac{1}{q(x+h)}$ --------------------------- (1.9)

And

Breadth of a Rectangle DOFE = Breadth of a Rectangle AGFE – Breadth of a Rectangle ABCD

Breadth of a Rectangle DOFE = AE– AD

Breadth of a Rectangle DOFE = $f(x+h) - f(x)$ -------------------- (2.0)

Thus,

Area of a Rectangle DOFE = Length of a Rectangle DOFE × Breadth of a Rectangle DOFE

Area of a Rectangle DOFE $=\frac{1}{q(x+h)}\times\left(f(x+h)-f(x)\right)$ --------------- (2.1) {Using (1.9) and (2.0)}

Step.4:

From Fig:1.3 , The sum of the areas of Rectangles BGOC and Rectangle DOFE can be find out by subtracting the Area of a Rectangle ABCD from the Area of a Rectangle AGFE .

Thus,

Area of a Rectangle AGFE – Area of a Rectangle ABCD = Area of a Rectangle DOFE + **Area of a Rectangle BGOC**

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{1}{q(x+h)}\times\left(f(x+h)-f(x)\right)+f(x)\left(\frac{1}{q(x+h)}-\frac{1}{q(x)}\right)$ ------------ (2.3)

{Using (2.1) and (1.8)}

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{f(x+h)}{q(x+h)}-\frac{f(x)}{q(x+h)}+\frac{f(x)}{q(x+h)}-\frac{f(x)}{q(x)}$

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{f(x+h)q(x)-f(x)q(x)+f(x)q(x)-f(x)q(x+h)}{q(x)q(x+h)}$

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{q(x)\left(f(x+h)-f(x)\right)+f(x)q(x)-f(x)q(x+h)}{q(x)q(x+h)}$

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{q(x+h)\left(f(x+h)-f(x)\right)-f(x)q(x+h)+f(x)q(x)}{q(x)q(x+h)}$

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{q(x+h)\left(f(x+h)-f(x)\right)-f(x)\left(q(x+h)-q(x)\right)}{q(x)q(x+h)}$ ----------- (2.4)

Since,

From Fig:1.3 , we have

Area of a Rectangle AGFE – Area of a Rectangle ABCD = $\frac{f(x+h)}{q(x+h)}-\frac{f(x)}{q(x)}$ ----------------- (2.5)

Using (1.5) and (1.2)

Thus equation (2.4) becomes

Area of a Rectangle AGFE – Area of a Rectangle ABCD $=\frac{q(x+h)\left(f(x+h)-f(x)\right)-f(x+h)\left(q(x+h)-q(x)\right)}{q(x)q(x+h)}$

$\therefore\frac{f(x+h)}{q(x+h)}-\frac{f(x)}{q(x)}=\frac{q(x+h)\left(f(x+h)-f(x)\right)-f(x+h)\left(q(x+h)-q(x)\right)}{q(x)q(x+h)}$ ---------------- (2.6) Using (2.5)

Thus,

The Derivative of a function, $y = \frac{f(x)}{q(x)}$ is given by

$$\frac{dy}{dx} = \lim_{h \to o} \frac{\left(\frac{f(x+h)}{q(x+h)} - \frac{f(x)}{q(x)}\right)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to o} \frac{\left(\frac{q(x)(f(x+h)-f(x)) \; - \; f(x)\,(q(x+h)-q(x))}{q(x+h)\,q(x)}\right)}{h}$$ {Using equation (2.6)}

$$\frac{dy}{dx} = \lim_{h \to o} \frac{\left(\frac{q(x)(f(x+h)-f(x)) \; - \; f(x)\,(q(x+h)-q(x))}{h}\right)}{q(x+h)\,q(x)}$$

$$\frac{dy}{dx} = \lim_{h \to o} \frac{\left(q(x)\frac{(f(x+h)-f(x))}{h} - f(x)\frac{(q(x+h)-q(x))}{h}\right)}{q(x+h)\,q(x)}$$

$$\frac{dy}{dx} = \lim_{h \to o} \frac{1}{q(x+h)\,q(x)} \lim_{h \to o} \left(q(x)\frac{(f(x+h)-f(x))}{h} - f(x)\frac{(q(x+h)-q(x))}{h}\right)$$

$$\frac{dy}{dx} = \lim_{h \to o} \frac{1}{q(x+h)\,q(x)} \left(q(x)\lim_{h \to o}\frac{(f(x+h)-f(x))}{h} - f(x)\lim_{h \to o}\frac{(q(x+h)-q(x))}{h}\right)$$

$$\frac{dy}{dx} = \frac{1}{q(x)\,q(x)} \left(q(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}q(x)\right)$$

$$\frac{dy}{dx} = \frac{\left(q(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}q(x)\right)}{(q(x))^2}$$

$$\therefore \frac{dy}{dx} = \frac{q(x)f'(x) - f(x)q'(x)}{(q(x))^2}$$ Hence Proved….

2. Product rules :

$$y = f(x)q(x)$$

$$\frac{dy}{dx} = f(x)q'(x) + q(x)f'(x)$$

Step :1 : -

<u>Finding the area of a Rectangle ABCD from Fig:1.3</u> :-

Consider $f(x)$ as the breadth of a rectangle ABCD and $q(x)$ as the length of a rectangle ABCD as shown in fig:1.4.

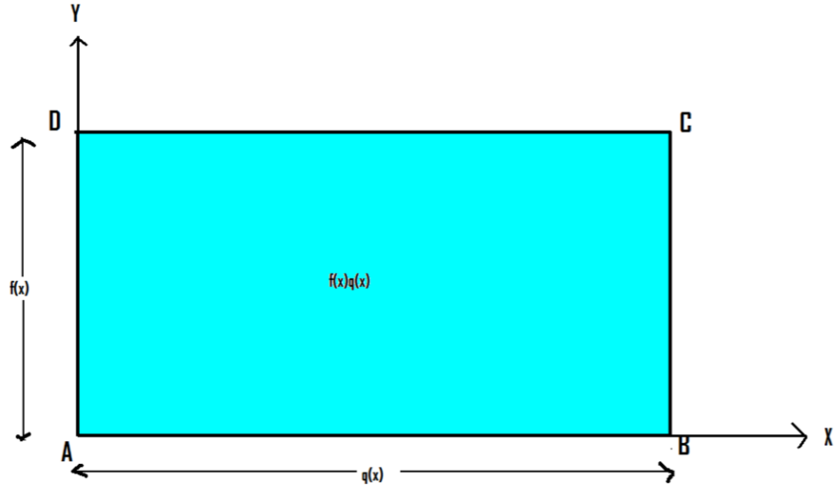

Fig:1.4

From a Rectangle ABCD (in Fig:1.4) we have ,

Length AB $= f(x)$ ------------------------------------ (2.7)
Breadth AD $= q(x)$ ----------------------------------- (2.8)

∴ Area of Rectangle ABCD = Length AB × Breadth AD
Area of Rectangle ABCD = $q(x) \times f(x)$

Area of Rectangle ABCD = $f(x)q(x)$ ------------------------- (2.9)

Step:2 :-
Introducing an increment h to both functions $f(x)$ and (x) :

If the function $f(x)$ has the increment h , then the function $q(x)$ will also have an increment h, as shown below in the fig.1.5

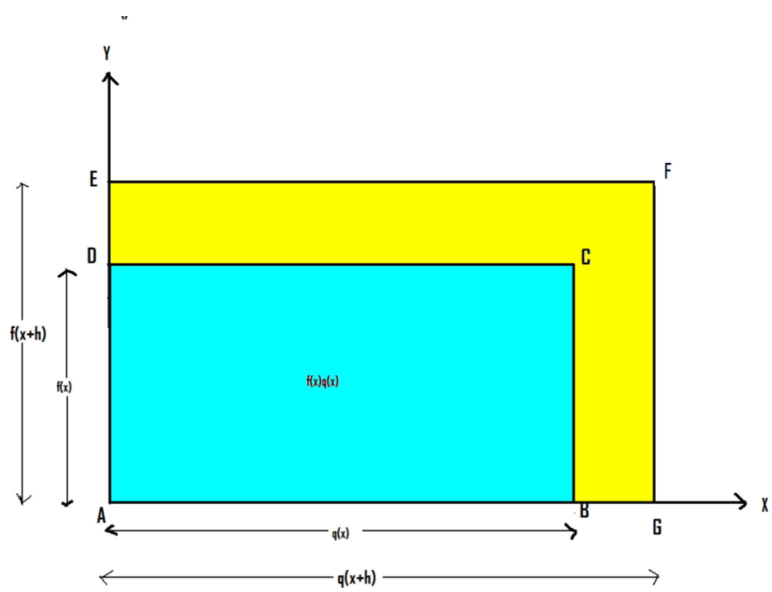

Fig:1.5

From a Rectangle AGFE (Fig:1.5) we have :

Length of Rectangle AGFE = AG
Length of Rectangle AGFE = $q(x+h)$ ------------------------ (3.0)

Breadth of Rectangle AGFE =AE
Breadth of Rectangle AGFE = $f(x+h)$ ------------------------- (3.1)

∴ Area of Rectangle AGFE = Length of Rectangle AGFE × Breadth of Rectangle AGFE
Area of Rectangle AGFE = $q(x+h) \times f(x+h)$ using (3.0) and (3.1)
Area of Rectangle AGFE= $f(x+h)q(x+h)$ ------------------------- (3.2)

Step.3:-

Finding the Area of the two new Rectangles (Rectangle BGCO and Rectangle DOFE) :-

After Producing a line OC as shown in Fig:1.6 , then the lengths , breadths and areas of the two new Rectangle BGOC and Rectangle DOFE can be find out .

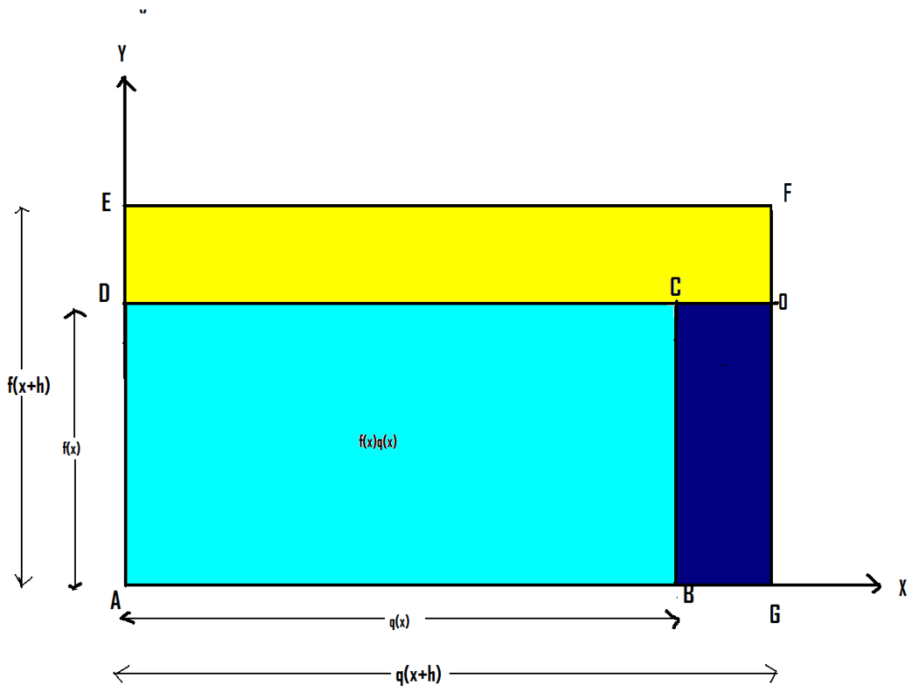

Fig:1.6

From the above fig:1.6 ,We have

Length of a Rectangle BGOC = Length of a Rectangle AGFE − Length of a Rectangle ABCD
Length of a Rectangle BGOC = Length AG − Length AB
Length of a Rectangle BGOC = $q(x+h) - q(x)$ -------------------------------- (3.3)

And

Breath of a Rectangle BGOC = Breadth of a Rectangle ABCD

Breath of a Rectangle BGOC $= f(x)$ -------------------------------- (3.4)

∴ **Area of a Rectangle BGOC** = length of a Rectangle BGOC × Breadth of a Rectangle BGOC

Area of a Rectangle BGOC $= \{(q(x+h) - q(x)\} \times f(x)$

Area of a Rectangle BGOC $= f(x)(q(x+h) - q(x))$ --------------------------- (3.5)

Also,

From the above fig:1.6 , We have

Length of a Rectangle DOFE =Length of a Rectangle AGFE

Length of a Rectangle DOFE = Length AG

∴ Length of a Rectangle DOFE $= q(x+h)$ -------------------------- (3.6)

And

Breadth of a Rectangle DOFE = Breadth of a Rectangle AAGFE – Breadth of a Rectangle ABCD

Breadth of a Rectangle DOFE = Breadth AE– Breadth AD

Breadth of a Rectangle DOFE = $f(x+h) - f(x)$ ------------------- (3.7)

Thus,

Area of a Rectangle DOFE = Length of a Rectangle DOFE × Breadth of a Rectangle DOFE

Area of a Rectangle DOFE = $q(x+h) \times \big(f(x+h) - f(x)\big)$ -------------- (3.8) {Using (3.6) and (3.7)}

Step.4:

The sum of the areas of Rectangles BGOC and Rectangle DOFE can be find out by subtracting the Area of a Rectangle ABCD from the Area of a Rectangle AGFE.

Thus,

Area of a Rectangle AGFE – Area of a Rectangle ABCD $= f(x+h)q(x+h) - f(x)q(x)$ ---------------- (3.9)

{Using (3.2) and (2.9)}

Since,

Area of a Rectangle AGFE – Area of a Rectangle ABCD = Area of a Rectangle BGOC + Area of a Rectangle DOFE

$\Longrightarrow f(x+h)q(x+h) - f(x)q(x) =$ Area of a Rectangle BGOC + Area of a Rectangle DOFE {Using (3.9)}

$$\Longrightarrow f(x+h)q(x+h) - f(x)q(x) = f(x)\big(q(x+h) - q(x)\big) + q(x+h) \times \big(f(x+h) - f(x)\big)$$

{Using (3.5) and (3.8) }

$$\Rightarrow f(x+h)q(x+h) - f(x)q(x) = f(x)\big(q(x+h) - q(x)\big) + q(x+h)\big(f(x+h) - f(x)\big) \quad \text{---------------} \ (4.0)$$

Thus,

The Derivative of a function, $y = f(x)q(x)$ is given by

$$\frac{dy}{dx} = \lim_{h\to o} \frac{(f(x+h)q(x+h) - f(x)q(x))}{h}$$

$$\frac{dy}{dx} = \lim_{h\to o} \left(\frac{f(x)\big(q(x+h)-q(x)\big)+q(x+h)\big(f(x+h)-f(x)\big)}{h}\right) \qquad \{\text{Using } (4.0)\}$$

$$\frac{dy}{dx} = \lim_{h\to o} \left(\frac{f(x)\big(q(x+h)-q(x)\big)}{h} + \frac{q(x+h)\big(f(x+h)-f(x)\big)}{h}\right)$$

$$\frac{dy}{dx} = \lim_{h\to o} f(x)\frac{q(x+h)-q(x)}{h} + \lim_{h\to o} q(x+h)\frac{\big(f(x+h)-f(x)\big)}{h}$$

$$\frac{dy}{dx} = f(x)\lim_{h\to o}\frac{q(x+h)-q(x)}{h} + \lim_{h\to o} q(x+h)\lim_{h\to o}\frac{\big(f(x+h)-f(x)\big)}{h}$$

$$\frac{dy}{dx} = f(x)\frac{d}{dx}q(x) + q(x)\frac{d}{dx}f(x)$$

$$\frac{dy}{dx} = f(x)q'(x) + q(x)f'(x)$$

Hence, Proved …….

3. **The Derivative of the function of a constant and a function is equal to the product of the constant and the derivative of the function.**

For, $\boldsymbol{y = cf(x)}$

$$\boldsymbol{\frac{d}{dx}\big(cf(x)\big) = c\frac{d}{dx}f(x)}$$

Step:1:- Finding the Area of a Rectangle ABCD from Fig:1.7

Consider c as the length of the Rectangle ABCD which is fixed and $f(x)$ as the breadth of a Rectangle ABCD as shown in Fig:1.7.

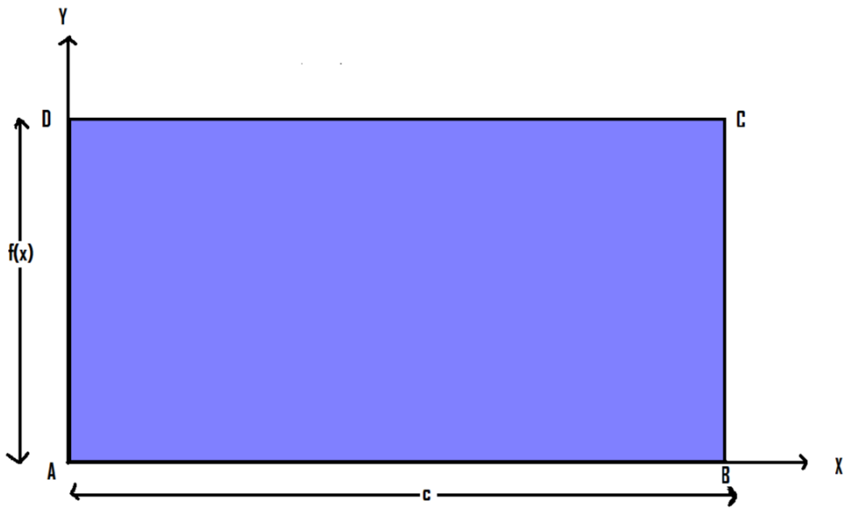

Fig:1.7

Therefore,

Length of a Rectangle ABCD = Length AB

Length of a Rectangle ABCD $= c$ ---------------------- (4.1)

And Breadth of a Rectangle ABCD = AD

Breadth of a Rectangle ABCD = $f(x)$ --------------- (4.2)

Hence,

Area of a Rectangle ABCD = Length of a Rectangle ABCD × Breadth of a Rectangle ABCD

Area of a Rectangle ABCD $= c \times f(x)$

$\therefore$ Area of a Rectangle ABCD $= c\, f(x)$ ------------------------ (4.3)

Step:2:- Finding the Area of a Rectangle ABEF from Fig:1.8

If the function $f(x)$ has the increment h , the value of the length of a Rectangle ABEF is equal to the length of a Rectangle ABCD as $\boldsymbol{c}$ is fixed (as $\boldsymbol{c}$ is a fixed value, therefore, its value cannot be change) as shown below in the fig.1.8 .

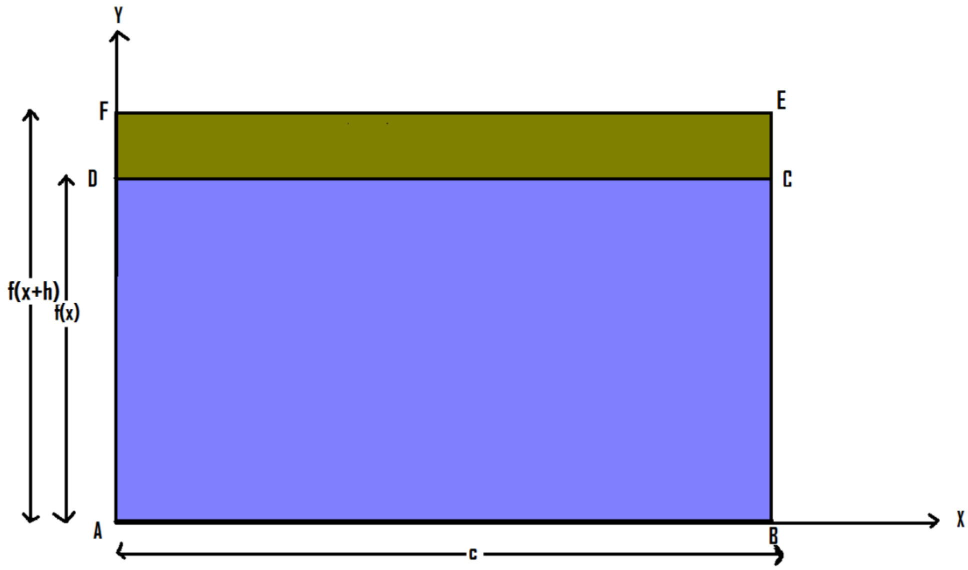

Fig:1.8

From fig:1.8, we have

Length of a Rectangle ABEF = Length of a Rectangle ABCD

∴ Length of a Rectangle ABEF = $\boldsymbol{c}$ ---------------- (4.4)

And,

Breadth of a Rectangle ABEF = Length AF

Breadth of a Rectangle ABEF = $f(x+h)$------------------- (4.5)

Then,

Area of a Rectangle ABEF = Length of a Rectangle ABEF × Breadth of a Rectangle ABEF

Area of a Rectangle ABEF = $f(x+h)\times \boldsymbol{c}$

∴ Area of a Rectangle ABEF = $\boldsymbol{c}f(x+h)$ ------------------------ (4.6)

Step:3:- Finding the Area of a Rectangle CDFE from Fig:1.8

From fig:1.8, we have

Length of a Rectangle CDFE = Length of a Rectangle ABCD

Length of a Rectangle CDFE = $\boldsymbol{c}$---------------- (4.7)

And,

Breadth of a Rectangle CDFE = Bread of a Rectangle ABEF − Breadth of a Rectangle ABCD

Breadth of a Rectangle CDFE = $f(x+h) - f(x)$ ------------ (4.8) { Using (4.5) and (4.0) }

Then,

Area of a Rectangle CDFE = Length of a Rectangle CDFE × Breadth of a Rectangle CDFE

Area of a Rectangle CDFE = $\boldsymbol{c}\times\{ f(x+h) - f(x) \}$ using (4.7) and (4.8)

∴ Area of a Rectangle CDFE = $\boldsymbol{c}\ \{ f(x+h) - f(x) \}$ ----------------------- (4.9)

Step:4:- Finding the Area of a Rectangle CDFE from Fig:1.8 by subtracting area of a Rectangle ABCD from area of a Rectangle ABEF .

From fig:1.7 , we Clearly we see that the area of a Rectangle CDFE can be find out by subtracting the area of a Rectangle ABCD from the area of a Rectangle ABEF

Thus,

Area of a Rectangle ABEF – Area of a Rectangle ABCD = Area of a Rectangle CDFE

$\boldsymbol{c}f(x+h) - c\ f(x)$= Area of a Rectangle CDFE on using (4.6) and (4.3)

∴ $\boldsymbol{c}\ f(x+h) - c\ f(x) = \boldsymbol{c}\ \{ f(x+h) - f(x) \}$ --------------- (5.0) on using (4.9)

Hence,

The Derivative of a function, $y = \boldsymbol{c}f(x)$is given by

$$\frac{dy}{dx} = \lim_{h\to o} \frac{\boldsymbol{c}f(x+h) - c\ f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h\to o} \frac{\boldsymbol{c}\ \{ f(x+h) - f(x) \}}{h}$$ {On using equation (5.0)}

$$\frac{dy}{dx} = \boldsymbol{c} \lim_{h\to o} \frac{f(x+h) - f(x)}{h}$$

∴

$$\frac{dy}{dx} = \boldsymbol{c}f'(x)$$

Proved.

CONCLUSION:-

This Project was a great way to help myself realize some things that I had not go through when I was as a studen. In this paper, the solution is solved by basic method in simple form and all the figures are made easy to understand.

I hope you will accept my project happily which I have made with calm and hard-working mind and sincere heart.

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